WORKING paper

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DSGE Nash: solving Nash Games in Macro Models

With an application to optimal monetary policy under monopolistic commodity pricing

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ABSTRACT

This paper presents DSGE Nash, a toolkit to solve for pure strategy Nash equilibria of global games in general equilibrium macroeconomic models. Although primarily designed to solve for Nash equilibria in DSGE models, the toolkit encompasses a broad range of options including solutions up to the third order, multiple players/strategies, the use of user-defined objective functions and the possibility of matching empirical moments and IRFs. When only one player is selected, the problem is re-framed as a standard optimal policy problem. We apply the algorithm to an open-economy model where a commodity importing country and a monopolistic commodity producer compete on the commodities market with barriers to entry. If the commodity price becomes relevant in production, the central bank in the commodity importing economy deviates from the first best policy to act strategically. In particular, the monetary authority tolerates relatively higher commodity price volatility to ease barriers to entry in commodity production and to limit the market power of the dominant exporter.

Keywords: DSGE Model, Optimal Policies, Computational Economics

JEL classification: C63, E32, E61

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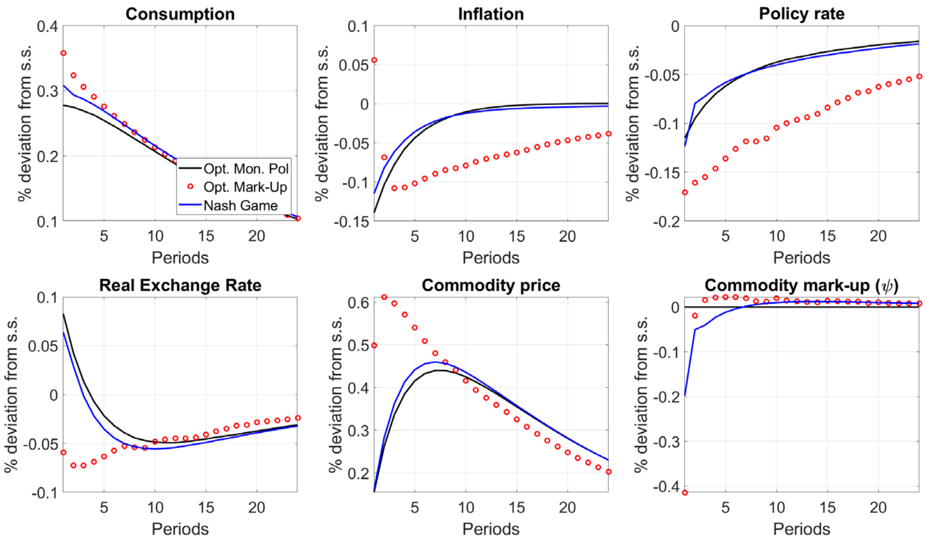
Non-Technical Summary

When individual agents choose strategically, they try to anticipate the actions of other players to maximise expected benefits. This behaviour might lead to outcomes that are inferior to what would be achieved via coordination, i.e. when agents maximise the global welfare of the economy. Studying strategic interactions allows to quantify these losses, detect the reasons why individual incentives might be inefficient and design policies that try to correct them. These mechanisms are relevant in macro models where some agents (e.g. countries) need to decide on the best policy to implement. However, modelling global games is significantly more complex in the context of macro and general equilibrium models. We present a new toolbox, DSGE Nash, that computes the solution of global games for a wide set of macroeconomic models. The toolkit envisages four main configurations that cover a wide range of problems. First, it can solve a Nash game between agents in general equilibrium models, with the model's remaining equations taken as constraints. Second, it can target a broad set of objective variables and it can be applied to different frameworks, including semi-structural and agent-based models. Third, when only one player is selected, DSGE Nash re-casts the problem into a standard optimal policy problem and solves it. Fourth, it can estimate models by moment or impulse response matching. All these functionalities are provided in an user-friendly environment that allows for the customization of both the model and the solution algorithm.

As an application example, we use DSGE Nash to study an open-economy model where a commodity importing country and a monopolistic commodity producer compete on a market characterized by barriers to entry. Rising commodity prices have been recently a main source of supply shocks, exacerbated by the monopolistic nature of commodity production. Against this background, the question of how policy makers should respond to commodity price swings has become prominent. We find that the degree of strategic competition between the producer and the importing country depends on the importance of the commodity in consumption and production. If the commodity share is low, both players have a dominant strategy requiring: i) strong core inflation targeting and no reaction to the commodity price by the commodity importer; ii) a somewhat lower mark-up setting for the monopolist. When the commodity is relevant for production, the central bank cannot follow the first-best policy because that would allow the exporter to extract higher rents, thus reducing domestic welfare in the importing economy. The central bank then knowingly chooses an alternative rule that tolerates higher volatility in commodity prices. Barriers to entry are hence lowered and the exporter faces more competition. To avoid the risk of new competitors entering the market, the exporter reduces its mark-up, which benefits the importing economy and ultimately increases welfare, despite a less efficient monetary policy rule.

Figure: Impulse responses to a positive TFP shock, commodity dependent economy



Notes: impulse responses for the parametrization reported in Table 3. IRFs are computed by solving the model at second order with pruning, a computational technique used to derive impulse responses for models solved at higher orders. Higher order terms in the dynamic system's solution might lead to explosive behaviour; pruning methods allow to eliminate those explosive paths and to derive impulse response functions (see for example Kim et al. (2008)).

DSGE Nash : résolution des jeux de Nash dans les modèles macroéconomiques

Avec une application à la politique monétaire optimale en cas de fixation monopolistique des prix des matières premières

RÉSUMÉ

Cet article présente « DSGE Nash », une boîte à outils permettant de résoudre des équilibres de Nash en stratégie pure pour des jeux globaux dans des modèles macroéconomiques en équilibre général. Bien qu'elle soit principalement conçue pour résoudre les équilibres de Nash dans les modèles DSGE, la boîte à outils englobe un large éventail d'options, y compris des solutions jusqu'au troisième ordre, des joueurs/stratégies multiples, l'utilisation de fonctions objectives définies par l'utilisateur et la possibilité de faire correspondre les moments empiriques et les IRF. Lorsqu'un seul joueur est sélectionné, le problème est reformulé comme un problème de politique optimale standard. Nous appliquons l'algorithme à un modèle d'économie ouverte dans lequel un pays importateur et un producteur monopolistique de matières premières sont en concurrence sur le marché des matières premières avec des barrières à l'entrée. Si le prix des matières premières devient pertinent dans la production, la banque centrale de l'économie importatrice s'écarte de la solution de premier rang en termes de politique économique pour agir stratégiquement. En particulier, l'autorité monétaire tolère une volatilité relativement plus élevée des prix des matières premières afin d'assouplir les barrières à l'entrée dans la production et de limiter le pouvoir de marché de l'exportateur dominant.

Mots-clés : modèle DSGE, politiques optimales, économie computationnelle

1 Introduction

"The complexity of the mathematical work needed for a complete investigation increases rather rapidly, however, with increasing complexity of the game; so that analysis of a game much more complex than the example given here might only be feasible using approximate computational methods." Nash (1951)

An important question in structural modelling is the determination of the "optimal policy" that is how agents or policy makers should respond to fluctuations in key macroeconomic aggregates in order to maximize their own objective function. This question has been widely discussed in the literature on monetary (Woodford (2003)) and fiscal (Chari et al. (1994)) policy, where the main concern is to determine the optimal response of either the central bank or the government to inflation, output and other observable variables so to maximize households' welfare. The solution to these problems needs to account for general equilibrium effects: the planner's decisions change incentives for other agents, thus altering equilibrium allocations and, in turn, feeding back into the planner's problem. Those second-round effects are generally large and enter in the optimizing agent's problem when setting the optimal policy.
The macro literature has proposed two ways to address this issue: i) "analitically", by optimizing the planner's objective function under the constraints given by the other equations in the model; ii) "numerically", by drawing values for the policy parameters and selecting those that give the highest value of the objective function. Both these methods generally target *equilibrium* allocations so to include the effect of the endogenous reaction of private agents. Moreover, in most applications, the objective variable (typically households' welfare if the optimizing agent is a public authority), needs to account for changes in both the level and volatility induced by the optimal policy (in the case of welfare, for example to consumption and labor) with the relative importance of the two depending on the specific policy or target variable chosen. For example, fiscal policies typically have a relevant first-order, i.e. level, impact while monetary policy affects welfare via a smoothing of the business cycle volatility.[1] These practical constrains can make simple solution methods not feasible. For instance, linear first-order solutions of macro models overlook the impact of volatility and, hence, cannot produce accurate metrics for policy analysis. In this regard, the existing literature provides two solutions. On the one hand, Benigno and Woodford (2012) and Benigno and Woodford (2004) show how to construct a linear-quadratic (LQ) approximation of the policy function. Notably, their methodology allows to set up second-

[1]In most macro models, the *equilibrium* level of the policy rate depends on households' inter-temporal preferences. Different monetary policy rules, instead, affect the way the central bank reacts to shocks, i.e. how much and how persistently the policy rate fluctuates around its long-run level.

order accurate welfare values based on their first-order approximations. While this approach is elegant and straightforward to implement, it is often very computationally demanding in larger models. The modeller needs indeed to compute a representation of the welfare function that includes the level and the interactions across all the endogenous variables. On the other hand, the model can be directly solved at the higher-orders and the second-order accurate mean of the objective variable can be computed via its state-space representation (Kim et al. (2008), Born and Pfeifer (2020)). This methodology, which accounts for the expected volatility of shocks in the *stochastic steady state*, has the advantage to be simple to implement once the solution of the model is automatized. Nonetheless, computing the state-space representation of large models at higher order can also be computationally demanding.

The quest for optimal policy is even more complicated when more than on player is involved. This is a typical problem in international macro models where several agents (typically central banks or governments) have to *contemporaneously* choose the optimal values of their policy functions. In these settings "*...it is assumed that each participant acts independently, without collaboration or communication with any of the others*" (Nash (1951)). In other terms, agents need to optimize their own policy functions by trying to anticipate what other players would do. They engage in a global game in pure strategies, where each player might decide to either avoid specific strategies in anticipation of the opponents' reaction or try to shift the costs of certain policies to her opponents. The final outcome, namely the Nash equilibrium of the game, is generally significantly different from the equilibrium under perfect cooperation. Clarida et al. (2002) and Banerjee et al. (2016) tackle this issue by explicitly solving the optimal control problem for each agent, so that she chooses the welfare-maximizing parameter of the policy function while keeping all other equations in the model as constraints to the choice problem[2]. The resulting first order conditions are then added to the model, including the solution for all the Lagrangian multipliers associated to the constraints, i.e. one for each player and each equation of the model. Under this approach, numerical derivatives are computed for each agent's policy variable problem and then added to the set of equilibrium conditions. The whole process can become fairly complex even in medium-sized models and it additionally requires that derivatives are continuous. Non-linearities in the model or in the policy function(s), due for example to occasionally biding constraints, cannot be accounted for by construction. Interestingly this suggests that a *game much more complex [...] might only be feasible using approximate computational methods*, as anticipated by Nash (1951).

Our algorithm, `DSGE Nash`, solves the same problem by means of numerical methods. In par-

[2] Bodenstein et al. (2019) has recently developed a toolkit to compute this solution.

ticular, the toolkit solves for the Nash equilibrium in pure strategies of policy games with N players and strategies. The solution algorithm first computes the payoff matrix of the game. Thereafter, it efficiently solves for the Nash equilibria of the game, i.e. it uses the payoff matrix to compute the combinations of strategies that are optimal for all players at the same time.

The deployment of numerical methods presents several advantages relative to numerical differentiation and constitutes an extension of the existing toolkit for the analysis of global games in macro models. First, in large models with more than two players, numerical solutions might indeed be faster and more feasible than numerical differentiation. In our toolkit, the state-space representation of the model is used to compute the *stochastic* steady state. Evaluating it for different convolutions of parameters could be faster than numerically differentiate a large set of equations, depending on the specific model under consideration. In addition, our solution method allows to target *any* policy function (including "optimal simple rules" such as the Taylor rule or a cyclical taxation rule) or objective variable (including a LQ approximation of the policy function, second or third moments of endogenous variables, impulse responses or any of their convolutions). Importantly, researchers might study optimal policy games using first-order approximation and LQ policy functions, that are significantly faster to compute numerically. Moreover, this allows to go beyond standard welfare or profit targeting and include other objectives for the planner's strategies such as the reaction after specific shocks or the volatility of specific variables. This could be relevant in the context of financial stability models whose focus is not on the behaviour of the system in the steady state, but rather on the reaction of the banking sector to tail shocks. Additionally, we can compute the Nash equilibria of the policy game even with non-linear models or a non-continuously differentiable policy function. Examples of non-continuously differentiable policy games are games where the central bank has to decide across several target variables (core or headline inflation for instance) or, more generally, when there are occasionally binding constraints. An additional strength point of `DSGE Nash` is that the algorithm can be applied to any model, even where a closed-form solution is not available. Moreover, `DSGE Nash` goes beyond optimization by providing a toolkit for moment (Ruge-Murcia (2012) and McFadden (1989)) and impulse response matching estimation (Guerron-Quintana et al. (2017)), thus significantly reducing the barriers to entry to those methods. Finally, our algorithm can be completely parallelized, which makes it possible to fully exploit the capabilities of modern server machines.

`DSGE Nash` features the following main configurations:

- **Policy equilibria in DSGE models (default):** the model is solved with Dynare[3],

[3] See Adjemian et al. (2020).

the algorithm first computes the payoff matrix via a second-order approximation and, then, the Nash equilibria of the game. If users can provide a LQ approximation of the objective function, options allow to solve the model at first order and use the LQ solution to construct the payoffs matrix.

- **Policy equilibria in macro models:** `DSGE Nash` lets users define a custom-made function to compute payoffs for each draw of the policy parameters. This can be easily done by executing the user-defined code in a dedicated `.m` file that substitutes the model solution by Dynare. This allows to apply our solution algorithm to all models where strategic interactions are relevant. As an example, in the next section we solve the Prisoner's dilemma game.

- **Standard optimal policy:** if only one player is selected, `DSGE Nash` automatically recasts the problem into a standard optimal policy problem. In this case the toolkit computes the value of parameters that maximize the objective function of the player. Even in this case, the baseline target is a second-order solution of the model through Dynare, but users can deploy any other user-defined code to compute the objective function.

- **Matching empirical moments:** the same set of algorithms used to compute the Nash equilibrium and optimal policies can be applied to the calibration of models by matching empirical data. Given the option, `DSGE Nash` can compute the values of a subset of the model's parameters to match empirical moments[4] or empirical impulse responses. User can specify the order of approximation at which model-generated moments or IRFs are computed as well as different weighting schemes in case of moment matching. These methods can be helpful when the size of the model prevents the efficient estimation with time series methods at higher-order.

Although our purpose is similar to that of Bodenstein et al. (2019), we depart from the latter in five respects. First, Bodenstein et al. (2019) solve for the Nash equilibrium by computing the numerical derivatives of the objective function of each player with all the other equations in the model and other agents' policy functions acting as constraints. `DSGE Nash`, on the other hand, uses simulated methods to fill up each player's payoff matrix and then finds the Nash equilibria of the game. Each approach presents benefits and costs. Computing numerical derivatives has the advantage of finding a closed-form solution for the policy problem. However, it can be very resource intensive, especially in large models with many variables. Moreover, Bodenstein et al. (2019)'s approach requires to add all the Lagrangian multipliers to the set of endogenous vari-

[4]See Ruge-Murcia (2012).

ables, which could rapidly increase the number of equations included in the model, especially in frameworks that involve multiple players, thus making it hard to find a solution (especially at higher orders). Conversely, our methodology does not require to add equations to the model and relies on simulated methods to compute payoffs for each combination of strategies. This can also be computational intensive for large models, but there are methods to address the dimensionality problem and the entire procedure is parallelizable. Second, our toolkit is more flexible in handling non-linear models, objective and policy functions. We allow agents to target anything that can be derived from a model, such as the volatility of specific variables or the response to a specific shock. This greatly extends possible applications of `DSGE Nash`. Related to this, by allowing for user-defined objective functions our toolkit can be easily adapted to models with non-linearities such as occasionally binding constraints which are becoming increasingly popular in the literature (e.g., Holden (2019) and Cuba-Borda et al. (2019)) or problems for which the objective function is not differentiable. Fourth, players' payoffs can be computed also for non-DSGE models or models not solved by Dynare. This last feature further enlarges the potential applications of our algorithm to models with heterogeneous agents or ABMs. Fifth, our toolkit goes beyond Nash games. For example, it re-writes the problem into a standard optimal policy problem if only one player is selected and allows to calibrate models via moment or impulse response matching, thus significantly reducing the barriers for researchers to approach this type of analysis. All in all, our toolkit differs considerably from Bodenstein et al. (2019) despite being motivated by the need to address similar research questions. We believe indeed that the two toolkits complement each other as they offer different solutions to the same problem. Depending on the specific research question and the practical difficulties faced, either of the two could be the most efficient solution and researchers can choose the one that best fits their needs case by case.

Against this backdrop, we provide a concrete example by applying `DSGE Nash` to the analysis of the strategic interactions between two agents, a commodity importer and a dominant commodity exporter, that compete on a commodity market featuring barriers to entry. If commodities account for a low share of the importer's consumption/production baskets, both players have a dominant strategy that requires strong core inflation targeting and no reaction to the commodity price on the part of the commodity importer[5] and a somewhat lower mark-up setting for the exporter. The problem becomes more complicated if commodities are relevant (30%) in the importer's economy. In that case the importer's central bank cannot follow the "first-best" policy because that would allow the exporter to extract higher rents. To avoid this scenario,

[5]See Blanchard and Galí (2007) and Filardo et al. (2020).

the central bank chooses an alternative rule that tolerates higher commodity prices and eases the entrance of new producers. To avoid the risk of new competitors entering the market, the exporter keeps the commodity price low, which benefits the importing economy and ultimately increases welfare, despite a less efficient monetary policy rule. This example highlights the importance of strategic interactions. The key element that allows for higher welfare is not the efficiency of the policy rule chosen, but the "discipline" imposed on the commodity exporter.
The remainder of the paper is structured as follows: Section 2 presents the strategic interaction problem in macro models and describe our solution algorithms; Section 3 applies our algorithm to a macro model of commodity prices where strategic interactions between commodity producers and consumers are important and policy functions are non-linear; Section 4 summarises the findings and concludes; Appendix C provides a user guide to our toolkit.

2 The Nash problem

In many macro models, agents are assumed to act rationally in that they optimize individual objectives by taking the decisions of other agents as given. What if agents acted also strategically? Would the equilibrium and the dynamics of the model change if agents *strategically reacted* to other agents' actions?
This question is particularly relevant when studying policy implementation. Consider for example the problem of two central banks that optimize their respective policy functions. As countries are connected by real and financial linkages, the central bank's choices in one economy would affect welfare in the other one, and vice versa. As a result, optimal domestic welfare can only be achieved by explicitly accounting for and reacting to the optimal policy in the foreign country. A similar problem arises when considering the taxation of global externalities: domestic policy makers (e.g., the government) might wait for their foreign counterparts to move first and impose a tax, so to reap the full benefits of a reduction in the externality without bearing any direct cost. Strategic competition between players might prevent from reaching the social optimum, see Ferrari and Pagliari (2021) for an example related to climate policies. Strategic dynamics play also an important role when the social planner and private agents interact in pricing setting. Absent perfect competition, indeed, the former might threaten a policy response to prevent producers from exploiting their market power and extracting excessively high rents. For instance, it has been documented that strategic interactions can explain the recovery path of the US economy after the Great Depression (Cole and Ohanian (2004)).
In these and other similar contexts, each agent internalizes the opponents' reaction functions into their optimization problem. Finding the solution under strategic interactions necessarily

requires to solve for the Nash equilibrium (equilibria) of the game played by agents, where all the other equations of the model act as constraints. The solution consists of identifying the best-response function of each player to all other players' strategies. The intersection(s) across all these functions is (are) the Nash equilibrium (equilibria) of the game (Dutta (1999)). Notably, the Nash equilibrium of the game might not lead to the optimal solution from a global standpoint. Agent-specific incentives or lack of credibility of players' commitment, in fact, might allow individual incentives to dominate the optimal equilibrium from a "social" perspective. This is a typical problem in models where agents need to coordinate on policies related to public goods that are non-excludable and non-rivalrous in consumption. More recently, such problems have been studied for the "consumption" of a typical non-excludable and non-rivalrous good: the environment.

2.1 Intuition

This section provides a simple example of the intuition underlying our solution method, which we deem useful to better understand the algorithm described in Section 2.2.

Consider one of most well-known static game, i.e. the prisoners' dilemma (Lacey (2008)) where two criminals (P1 and P2) need to decide whether to confess the crime they committed (C_i) or to remain silent (S_i).[6] If they both confess, $\{C_1, C_2\}$, they will be both sentenced to 5 years of prison. If they both stay silent, $\{S_1, S_2\}$, the persecutors will not have a full account of their deeds and they will be sentenced to just 2 years of prison each. If only one of them confesses $\{C_1, S_2\}$, she will be able to shift the blame to her associated and will be set free, while the other prisoner will be severely punished with a 10-year prison sentence. Each player, therefore, has two pure strategies $\{C_i, S_i\}$ with payoff matrix as reported in Table 1.

The cooperative solution, $\{S_1, S_2\}$, maximizes the *joint* welfare of the criminals, as they would only face 2 years of prison each, 4 in total. However, individual's incentives and the lack of a credible commitment might prevent the players from reaching that solution. Each criminal knows that if she stays silent, her opponent will have the incentive to confess. The final payoff of that ending would be 10 years for the "silent" criminal and 0 for the "confessing" one, which is clearly worse from the perspective of the player who decides to stay silent. On the other hand, by confessing she will serve the least amount of years in prison independently on whether the other criminal will confess (5 instead of 10) or stays silent (0 instead of 2). Each prisoner knows this and endogenously decides to confess, independently of what the other will do. The Nash equilibrium of the game is then $\{C_1, C_2\}$, thus each will serve 5 years of prison, 10 in total,

[6] $i \in \{1, 2\}$.

Table 1: The prisoner's dilemma

		P2	
		C_2	S_2
P1	C_1	$(\underline{5}, \underline{5})$	$(\underline{0}, 10)$
	S_1	$(10, \underline{0})$	$(2,2)$

Notes: payoffs for each players are reported in parentheses. Values underlined in blue (red) represent the optimal response of P1 (P2) for a given strategy of the other player. The Nash equilibrium is given by the intersection of the best response functions, i.e. when both values in the payoff matrix are underlined. This game is solved with `DSGE Nash`, the code is provided in the `PrisonersDilemma.m` example.

much more than the "cooperative" scenario. This popular game gives the basic intuition as to why the equilibrium under strategic behaviour might be completely different from the optimal solution of a "global planner". In macro models, without credible commitments, countries face the same trade-off. They expect that other players might deviate from the social optimal strategy if individual incentives are strong enough, i.e. if by deviating they can seize sufficiently high gains. Because of that, inefficient equilibria are often reached.[7]

This simple game is also useful to highlight the mechanics behind the identification of a Nash equilibrium. A strategy vector $\sigma = \{s_i\}_{i=1}^{N}$ is an equilibrium iff it contains strategies that are optimal for all players $i = 1, \ldots, N$ at the same time. In other terms, the strategy $s_j \in \sigma$ needs to be optimal for player j given the strategies played by all the other players, $s_{-j} \in \sigma$. If this is the case, player j has no incentive to play anything else than s_j when the other $N-1$ players play s_{-j}. If this holds for all the N participants in the game, then σ is a Nash equilibrium.

Computing all the possible Nash equilibria in the game requires the identification of each player j's best response to any combination of the other players' strategies (s_{-j}). The vector of Nash equilibria is given by the intersection(s) across the players' best response functions. Strategies in the intersection(s) are, indeed, optimal for all players at once, given the actions of other players. In the prisoners' dilemma example, P1's best response to both C_2 and S_2 is playing C_1. The vector of best response strategies for P1 is therefore: $V_1 = \{(C_1, C_2)\, ;\, (C_1, S_1)\}$. With a similar logic, the vector of best responses for P2 is $V_2 = \{(C_1, C_2)\, ;\, (S_1, C_2)\}$. The only intersection between the two vectors, $\{C_1, C_2\}$, is the Nash equilibrium of the game. The same logic can be applied to computing pure-strategy equilibria in more complex games.

[7] These problems have been extensively studied for example in the course of arms race games, see Baliga and Sjöström (2004).

2.2 Solution algorithm

Consider now the general problem of player i who wants to maximise the objective variable W with respect to the instrument vector τ_i, conditional on the response of other players $-i$ and a set of constraints. In a structural model, such constrains are typically the structural equations and the shock processes. W can be any outcome of the model. Although in most applications W is the steady-state of some endogenous variables, our toolkit allows for any output of the model to be used as objective, including steady-state volatility or impulse responses.

Player i's problem can be written more formally as:

$$\begin{aligned} &\max_{\tau_i} E_0(W_i)|\tau_{-i} \\ &\text{s.t. } E_0(\Omega(x_{t+1}, x_t, x_{t-1}, \eta_t)) = 0, \end{aligned} \tag{2.1}$$

where x and η are vectors of endogenous variables and shocks respectively and $\Omega(\cdot)$ defines the policy function of the model which acts as constraint. Given a specific draw of the instrument of player i (τ_i) and the instruments of her opponents (τ_{-i}), the problem in Equation (2.1) can be solved by first computing the model equilibrium as informed by $\Omega(\cdot)$ and, then, by extracting the corresponding $E(W_i)|_{\boldsymbol{\tau}=(\tau_i,\tau_{-i})}$. The Nash equilibrium (equilibria) of the game is (are) given by the intersection(s) across the optimal response functions of the players, i.e. the intersection(s) across the sequences of strategies $\{\boldsymbol{\tau}\} = \{\tau_i|\tau_{-i}\}_{i=1}^N$ that solve Equation (2.1) for each player i. After obtaining a value of the objective $E(W_i)$ for each i and each combination of strategies $\boldsymbol{\tau} = (\tau_i, \tau_{-i})$, the Nash game is solved as follows:

1. for each combination of opponent's strategies, τ_{-i}, we compute the optimal response of i, i.e. the value of τ_i that maximises the objective $E(W_i)$ given τ_{-i} and $E_0(\Omega(\cdot))$. This is the optimal response of i to the strategies of the other players ($-i$) given the model's constraints;

2. we repeat the previous step for all combinations of strategies τ_{-i} and collect the optimal responses of i in the vector $\vec{\tau_i^*}|-i$, which is the optimal response function of player i to each τ_{-i};

3. we construct the vector of strategies $\mathcal{T}_i^*$ that matches the optimal response of i ($\vec{\tau_i^*}|-i$) with the strategies of other players $-i$ for which $\vec{\tau_i^*}|-i$ is the best response, i.e. $\mathcal{T}_i^* = \{(\tau_i^*, \tau_{-i})_j\}_{j=1}^S$ where $j \in [1, S]$ is the sequence of possible strategy combinations by players $-i$. In the Prisoner's dilemma these vectors would be $\mathcal{T}_1^* = \{(C_1, C_2), (C_1, S_2)\}$ and $\mathcal{T}_2^* = \{(C_1, C_2), (S_1, C_2)\}$;

4. we repeat steps (1)-(3) for each $i = 1, \dots, N$ player;[8]

5. the Nash equilibria of the game are given by the intersections of the vectors $\mathcal{T}_i^*$, $i = 1, \dots, N$

Steps (1) to (5) become computationally heavier the higher the number of players. We exploit an important property of $\mathcal{T}_i^*$, reported in Lemma 2.1, to solve the problem more efficiently.

Lemma 2.1. *All Nash equilibria are contained in the best response vector of one player.*

Proof. The best response vector of player i is defined as $\mathcal{T}_i^* = \{(s_i^*, s_{-i})_j\}_{j=1}^S$, with $j \in [1, S]$ begin the possible combinations of strategies by player i (s_i) and strategies of other players $-i$ (s_{-i}).

If the vector $\sigma = \{s_i\}_{i=1}^N$ is a Nash equilibrium, by definition, $s_i^* \in \sigma$ and $s_i^* \in \mathcal{T}_i^*$. Since σ is a Nash equilibrium of the game, $s_{-i} \in \sigma$ are also equilibrium strategies for the other players; therefore $(s_i^*, s_{-i}) \in \sigma$ and $(s_i^*, s_{-i}) \in \mathcal{T}_i^*$, because given s_{-i} player i optimally chooses strategy s_i^*.

Consider now the case of one hypothetical Nash equilibrium, (s_i', s_{-i}^*), that does not belong to $\mathcal{T}_i^*$. If $s_i' \notin \mathcal{T}_i^*$ then there is another strategy for i that gives higher payoffs to player i given s_{-i}^*. If that is true, i will not play s_i' in response to s_{-i}^*. But then (s_i', s_{-i}^*) is not a Nash equilibrium because i has incentives to deviate from s_i'. This reasoning is symmetric and applies to $-i$ players as well. If s_i is indeed a Nash equilibrium of the game, $(s_i, s_{-i}) \in \sigma$ must belong to $\mathcal{T}_i^*$ as s_i is both a best response and a Nash equilibrium strategy. $\square$

From Lemma 2.1 it follows that it is not necessary to repeat all steps (1)-(5) for each player, but one can simply look at the best response function of one player and check whether there is a specific vector of strategies that is optimal for all players at the same time. As players might have vector of strategies with different dimensions, one can strategically choose to consider only the vector of best responses with the shortest length. `DSGE Nash` relies on a grid search across strategy parameters values to populate the pay-off matrix.

2.3 Single player case & matching of empirical data

The same computational tools developed to find the Nash equilibria of policy games can be easily adapted to a single player setting or to calibrate key parameters of the model by matching empirical moments, see Ruge-Murcia (2012). `DSGE Nash` allows for both configurations and the algorithms are briefly presented in this section.

[8]In practice `DSGE Nash` solves the model over a grid of possible strategies for each player, which is supplied by the user.

Single player. When there is only one player, the game collapses to a standard optimal policy exercise, see Woodford (1999) and Sbordone et al. (2010). As only one agent (i) optimally chooses, the vector of constraints τ_{-i} is empty and the optimal problem in Equation (2.1) boils down to:

$$\begin{aligned} &\max_{\tau_i} E_0(W_i) \\ &\text{s.t. } E_0(\Omega(x_{t+1}, x_t, x_{t-1}, \eta_t)) = 0. \end{aligned} \tag{2.2}$$

Therefore, there are no strategic interactions and the single player chooses policy parameters in order to maximize the objective function $E(W_i)$. This problem can be solved numerically as in Born and Pfeifer (2020) as follows:

1. draw a value for the policy parameters τ_i;

2. solve the model conditional on the draw of τ_i and the constraint $E_0(\Omega(\cdot))$ to compute the objective $E(W_i)$;

3. repeat the steps (1)-(2) until a maximum is reached. Maximisation can be achieved with a search algorithm or by finding the highest objective $E(W_i)$ over a grid of possible values selected for τ_i.[9]

Matching empirical data. DSGE models can be calibrated to match empirical moments of key economic variables or impulse responses from empirical models. Matching empirical moments is a popular estimation tool for DSGE models and is particularly useful when time series methods are not feasible. That is the case, for example, of large models solved at higher orders, that cannot be estimated via standard Bayesian or frequentist approaches because of the long time needed to compute the likelihood function. Ruge-Murcia (2012), in particular, shows that moment matching can deliver accurate parameter estimates of non-linear models.[10] Our algorithm, however, allows to perform a simple matching of empirical time series. Given a subset of the model's parameters, θ, `DSGE Nash` searches for values that minimize the distance between empirical (s^e) and model generated moments (s^m). Formally:

$$\min_{\theta}(s^e - s^m)'\mathcal{W}(s^e - s^m), \tag{2.3}$$

[9] `DSGE Nash` allows for both options. The choice between the two depends on the practical implementation. Minimization algorithms might converge to a local minimum for problems with high non-linearities in the gradient relative to τ_i. In those cases a grid search algorithm should be preferred.

[10] The reader can refer to Fernández-Villaverde and Guerrón-Quintana (2020) for a comprehensive overview of these estimation methods.

where $\mathcal{W}$ is a weight matrix. s^e can include the volatility of endogenous variables, their correlations or some target mean values.[11]

Equation (2.3) can be also used to calibrate deep parameters by impulse responses (IRFs) matching. In this case, parameters are selected such that model-implied IRFs match those of an empirical model, usually computed via VARs or local projections. Instead of single moments, IRFs matching minimizes the squared cumulative distance between the empirical and the model-based IRFs. In other terms, when impulse response matching is selected, $s^m \equiv \{y^m_{\epsilon,k}\}_{k=0}^K$ and $s^e \equiv \{y^e_{\epsilon,k}\}_{k=0}^K$ with $y^e_{\epsilon,k}$ being the empirical IRF of variable y to shock ϵ at horizon k and $y^m_{\epsilon,k}$ the model-based equivalent.

Both minimization problems are solved as follows:

1. draw a value for the parameter vector θ;
2. solve the model and compute model-based moments (or impulse responses);
3. compute the distance between model-based and empirical moments and update the draw;[12]

Equation (2.3) can be solved by standard minimization algorithms[13] or by grid search. `DSGE Nash` uses the former approach, by allowing the user to select specific minimization algorithms.

Final considerations. Our solution algorithm relies on the definition of an objective variable W_i or model's moments s^m. In the standard setting of `DSGE Nash`, these variables are derived from the solution (at *any* order) of a DSGE model solved using Dynare. However, users can compute them in alternative ways by providing the relevant calculations in an editable `.m`[14] file, which replaces the call of Dynare in the toolkit. In this way it is possible to target non-standard outcomes of the solution provided by Dynare and to use alternative solution algorithms or modelling frameworks. Details on how to invoke the alternative solution files and examples are provided in Appendix C. Similarly, `DSGE Nash` allows for different numerical minimization algorithms and options for populating the payoff matrix, for example by excluding specific draws of the parameters. This is particularly useful when the players' policy function is non-continuous along all its dimensions. Consider, for instance, the problem of a central bank that needs to strategically choose to respond to either core (ϕ_{core}) or headline (ϕ_{head}) inflation. This condition can be simply implemented by excluding all draws for which both $\phi_{core} > 0$ and

[11]The targets of highest interest are typically the volatility of inflation rates and output components, their correlation, the mean of spreads and consumption to output ratios.

[12]In the case of IRFs matching, the distance $s^e - s^m$ is the cumulative distance over the impulse response horizon.

[13]See Sims (1999) and Nocedal et al. (2014).

[14]See the option `userdefinedfunction.m` in the Appendix.

$\phi_{head} > 0$. Appendix C describes how to set alternative algorithms and how to specify those exclusion conditions.

3 Application to an open-economy model with commodity price competition

We use the toolkit to analyze the strategic interactions between two countries, an oil importer and an oil monopolistic exporter, within an open economy model with fringe commodity producers.[15] Rising commodity prices have been recently a main source of supply shocks, exacerbated by the monopolistic nature of commodity production. Against this background, the question of how policy makers should responds to commodity price swings has once again become prominent. To answer this question, however, it is crucial to understand how agents *strategically* interact: a monopolistic commodity producer will not rise prices to the point of forcing central banks to hike rates to counter inflation, thus inducing a contraction of demand and a fall of profits. Additionally, commodity producers will avoid excessively high prices to keep fringe competitors out of the market. This concern is particularly relevant in the oil market where monopolists are threatened by shale oil producers that enter the market only if prices are sufficiently high, see Kilian (2016). Commodity importing countries, instead, are tempted to compress commodity prices to keep production costs low. This policy reduces competition in commodity production, allowing exporters that enjoy lower production costs to consolidate their dominant position. We try to analyse this fundamental trade-off through the lens of our model.

In particular, we investigate how central banks and dominant commodity exporters should calibrate their policies in an environment of strategic interactions. Specifically, commodity producers have monopoly power because they control the supply of a scarce resource that is used by consumers and firms in other countries. They leverage on such monopoly power by setting the price mark-up on the commodity in order to maximize individual objectives (i.e. profits). Central banks in the commodity-importing block, instead, set monetary policy to maximise households' welfare. These agents, the central bank and the commodity monopolist have diverging objectives, but their choices affect each other. For example, an higher mark-up implies higher commodity prices and domestic inflation, which in turn triggers a tighter monetary policy, thus leading to a lower welfare. Meanwhile, a tighter monetary policy reduces demand,

[15] As discussed more in details later, fringe producers produce commodities as well, but have higher costs than the monopolistic country. For this reason, they engage in production only if the price of the commodity is higher than a threshold.

including for the commodity, which dampens the exporter's profits. Moreover, if the prevailing commodity price is sufficiently high, more fringe producers enter the market, reducing the market power and the profits of the dominant exporter. The dominant exporter might then act preemptively, by avoiding excessively high mark-ups to block the entrance of fringe producers. We use our `DSGE Nash` to solve the strategic game between the central bank and the commodity exporters, a task made complicated by the presence of relevant non-linear features. We find that that there are two possible configurations for the final equilibrium, which depend on the central bank's tolerance for inflation and on the relevance of commodities in production.

3.1 The model

Our baseline framework is built on Nakov and Pescatori (2010) and Filardo et al. (2020).[16] There are three blocks: a monopolistic commodity-exporter country (CEC), a commodity-importer country (CIC) and a fringe of commodity producers (CF) that supply commodity in perfect competition only if commodity prices are sufficiently high. The underlying assumption is that the CF has higher production costs relative to the CEC, but can flexibly increase or reduce production. Therefore, CF starts producing only when the commodity price is sufficiently high, while they stop production when the price falls below a specific threshold. These assumptions are inspired to the oil market and the rise of shale oil production.[17] As detailed in Filardo et al. (2020), households in the CIC consume the commodity and final consumption goods, supply labor to firms and save through one-period safe assets. First order conditions for the household's problem are:

$$
\begin{aligned}
1 &= \beta \frac{R_t}{\pi_{t+1}} \frac{\exp\left(e^C_{t+1}\right)}{\exp\left(e^C_t\right)} \frac{C_t}{C_{t+1}} \\
w_t &= C_t L_t^{\nu} \\
C_{Y,t} &= (1-\gamma)\frac{P_t}{P_{Y,t}} C_t \\
\mathcal{M}_{Q,t} &= \gamma \frac{P_t}{P_{Q,t}} C_t
\end{aligned}
\tag{3.1}
$$

where C is aggregate[18] consumption, L labor supply, e^C a preference shock, R the nominal interest rate, π headline inflation, w the real wage, C_Y consumption of domestic goods, $\mathcal{M}_Q$

[16]For brevity we report in this section only the main equilibrium conditions. The full model is described in Appendix B.

[17]See Bjørnland et al. (2021), Farrokhi (2020) and Kilian (2017).

[18]Domestic goods and commodities are bundled together into final consumption through the aggregator $C_t = (C_{Y,t})^{1-\gamma} (\mathcal{M}_{Q,t})^{\gamma}$. Notice also that the marginal utility of consumption, which is equal to the Lagrangian multiplier associated to household's problem, is also equal to $\frac{1}{Ct}$.

commodity demand by households, P the CPI price index[19], P_Y the price of consumption goods and P_Q the commodity price. β is the discount factor, ν defines the elasticity of labor supply and $\gamma \in (0,1)$ is the share of commodity in the consumption basket. Aggregate consumption then depends on preferences, γ, and the relative prices of commodities and consumption goods, P_Y and P_Q. If the price of the commodity increases, households re-balance between commodity and consumption goods but there is a dead-weight loss (due to income and substitution effects) in aggregate consumption, which in turn leads to welfare losses.

Firms in the CIC use a production technology that combines labor and the commodity as follows: $Y_t = \exp(A_t)(L_t)^{1-\alpha}(\mathcal{M}_{Y,t})^{\alpha}$. First order conditions for the firm's problem are:

$$\begin{aligned} MC_t &= \frac{W_t^{1-\alpha} P_{Q,t}^{\alpha}}{\exp(A_t)(1-\alpha)^{1-\alpha}\alpha^{\alpha}} \\ L_t &= (1-\alpha)\frac{MC_t}{w_t} Y_t \\ \mathcal{M}_{Y,t} &= \alpha \frac{MC_t}{P_{Q,t}} Y_t \end{aligned} \tag{3.2}$$

where MC is the marginal cost of production, Y total output, A a total factor productivity shock and $\mathcal{M}_Y$ is the commodity demand in production. In addition, $\alpha \in (0,1)$ is the share of commodity in the production function. If the commodity price increases, firms substitute the commodity with labor in the production function, which pushes marginal costs and prices up.

Final prices are set by monopolistic firms with some degree of market power. We follow the Calvo formalism and allow these firms to update their prices only with probability $1-\theta$, see Calvo (1983). Aggregate prices can be written in the following recursive form:

$$\begin{aligned} \mathcal{D}_t &= \frac{Y_t}{C_t} + \theta\beta E_t\left(\pi_{core,t+1}^{\epsilon-1}\mathcal{D}_{t+1}\right) \\ \mathcal{N}_t &= \frac{Y_t}{C_t}\frac{\epsilon}{\epsilon-1}MC_t\frac{Y_t}{C_t} + \theta\beta E_t\left(\pi_{core,t+1}^{\epsilon-1}\mathcal{N}_{t+1}\right) \\ \theta\pi_{core,t+1}^{\epsilon-1} &= 1-(1-\theta)\left(\frac{\mathcal{N}_t}{\mathcal{D}_t}\right)^{1-\epsilon} \\ \Delta_t &= (1-\theta)\left(\frac{\mathcal{N}_t}{\mathcal{D}_t}\right)^{\epsilon} + \theta\Delta_{t-1}\pi_{core,t}^{\epsilon}, \end{aligned} \tag{3.3}$$

where $\pi_{core,t} = \frac{P_{Y,t}}{P_{Y,t-1}}$ is core inflation, Δ is the price dispersion term and ϵ the elasticity of substitution across different varieties of consumer goods. Total commodity demand $\mathcal{M}_t$ is the sum of the commodity used in consumption and production:

$$\mathcal{M}_t = \mathcal{M}_{Y,t} + \mathcal{M}_{C,t} \tag{3.4}$$

[19] As implied by the consumption aggregator, the CPI index is defined as $P_t = (P_{Y,t})^{1-\gamma}(P_{Q,t})^{\gamma}$.

Monetary policy follows a Taylor rule:

$$R_t = \frac{1}{\beta} \left(\Pi_{core,t}\right)^{\phi_{core}} \left(\Pi_{head,t}\right)^{\phi_{head}} \left(\Pi_{com,t}\right)^{\phi_{com}} \left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_Y} \exp e_t^R \tag{3.5}$$

In this model a higher commodity price P_Q entails an income loss for consumers and higher costs of production for firms that in turn increase the price of consumption goods. The combined effects of the two is detrimental for welfare, whose reduction might be exacerbated by the monetary policy reaction function. If the central bank follows a strong inflation target, in fact, it will aggressively react to reduce inflation after a commodity price shock triggering a further contraction of output and consumption. However, the commodity exporter is not indifferent to these dynamics. If commodity prices increase and the central bank in the CIC raises rates, demand for the commodity will fall, thus reducing profits.

Turning to commodity production, CEC exports the commodity to the CIC and imports final consumption goods from the latter. The CEC sets the commodity price as a mark-up on the marginal cost in order to maximise profits:

$$P_{Q,t} = \Psi_t Z_t^{-1} \tag{3.6}$$

where Z is the marginal cost of production that is assumed to be exogenous.[20]. The mark-up Ψ_t depends on the supply of commodity from the dominant producer ($\mathcal{M}^{cec}$) and from the fringe ($\mathcal{M}^{cf}$):

$$\Psi_t = \psi_t \left(1 + \frac{\mathcal{M}_t^{cec}}{2\mathcal{M}_t^{cf}}\right) \tag{3.7}$$

where ψ is a policy parameter for the commodity producer to set the mark-up level. In the baseline calibration of the model ψ is constant and equal to 1. Profits for the CEC are:

$$\Pi_t = \mathcal{M}_t^{cec} \left(\Psi_t P_{Q,t} - Z_t^{-1}\right). \tag{3.8}$$

Under the assumption of perfect markets, the real exchange rate s is defined as the ratio of marginal utility of consumption between the CIC and the CEC. We assume that commodity prices are expressed in the currency of the CIC, which is consistent with commodity markets being largely settled in one single currency, namely the US dollar. We implicitly also assume that the fringe production is equally traded in the currency of the importer. Because of these assumptions, an appreciation of the CIC currency has ambiguous effects. On the one hand, it increases the value of profits of the commodity exporter, while, on the other hand, it reduces

[20]The marginal cost follows the process: $Z_t = (Z_{ss})^{1-\rho_z} (Z_{t-1})^{\rho_z} \varepsilon_t^z$, with $\varepsilon_t^z \sim N(0, \sigma_z)$.

the purchasing power of consumers in the CEC that consume goods produced by the CIC.
The commodity-fringe production is composed by atomistic producers with i.i.d. volatility around the marginal cost of production (ω_i) uniformly distributed in the interval $[a, b]$. These producers enter the market if the commodity price is higher than a threshold $\bar{P}_{Q,t}$.[21] Defining the mass of fringe producers active in period t as Ω_t, the total supply of the fringe is:

$$\mathcal{M}_t^{cf} = \Omega_t P_{Q,t} Z_t di \tag{3.9}$$

The model's equilibrium conditions can be compactly written as:

$$E_t \Omega(x_{t-1}, x_t, x_{t+1}, \eta_t) = 0 \tag{3.10}$$

with x and η being the vectors of endogenous variables and shocks respectively, while $\Omega(\bullet)$ is the policy function. Parameters are calibrated as reported in Table 2.
From the CEC's viewpoint, increasing the mark-up ψ has opposite effects. On the one hand, higher ψ increases the commodity price, leading to higher expected profits. However, on the other hand, higher commodity prices lead to higher inflation in the CIC, less commodity demand and a depreciation of the domestic currency. Moreover, the higher is the commodity price the more fringe producers enter the market and the lower is the monopoly power of the dominant exporter, see Equation (3.7). Hence, it is not clear *a priori* whether it is preferable for the CEC to set a high or a low ψ. From the prospective of the CIC, instead, a higher mark-up implies tighter monetary policy and, then, lower welfare. Policy makers in the CIC, therefore, would prefer an equilibrium with a low commodity price, which can be achieved either through a low value of the mark-up parameter ψ or a large share of fringe producers. When commodity prices are higher, instead, the central bank in the CIC is forced to tighten the policy rate to lower inflation. This, in turn, appreciates the exchange rate, increases the cost of imports for the CEC and reduces commodity demand. As anticipated, as profits for the CEC are expressed in the CIC currency, the effects of an exchange rate appreciation are not clear *ex ante*. Nonetheless, the monetary policy channel, i.e. higher rates in response to higher commodity price, might work as a "discipline mechanism" limiting the incentive for the CEC to use its monopoly power for two reasons: i) imports of consumption goods for the exporter become more expensive; ii) the contraction of the CIC economy might reduce the extensive margin of profits for the commodity producer.

[21] We assume that the threshold is time-varying and exogenous to capture volatility and time-variation in the cost function of these firms.

Table 2: Calibration

Parameter	Description	Value	Parameter	Description	Value
ψ_{ss}	Commodity Markup	1	ρ_Z	AR com. supply	0.5
γ	Commodity share in CPI	0.1	ρ_A	AR TFP	0.95
α	Commodity share in production	0.1	ρ_C	AR cons. preference	0.5
ν	Labor elasticity	0.5	ρ_R	AR mon. policy	0.5
ϵ	Elasticity of substitution	7.67	ρ_Ω	AR fringe supply size	0.5
β	Discount factor	0.99	σ_Z	Std com. supply	0.01
θ	Calvo pricing	0.75	σ_A	Std TFP	0.01
ϕ_{head}	Reaction to headline inflation	0.00	σ_C	Std cons. preference	0.01
ϕ_{core}	Reaction to core inflation	1.50	σ_Ω	Std fringe supply	0.01
ϕ_{com}	Reaction to commodity inflation	0.00	σ_R	Std mon. policy	0.01
ϕ_y	Reaction to output growth	0.00			

Notes: parameter values are taken from Nakov and Pescatori (2010) and Filardo et al. (2020). χ is calibrated to normalize labor supply to one in the steady state. Values for ψ_{ss}, ϕ_{head}, ϕ_{core}, ϕ_{com}, ϕ_y refer to the baseline calibration without optimal policy.

3.2 The policy game

In this model the CIC's central bank and the commodity monopolist have different -and conflicting- objectives. The central bank seeks to set monetary policy, i.e. choose the parameters ϕ_{head}, ϕ_{core}, ϕ_{com}, ϕ_y, to maximize households' welfare, whereas the commodity-exporter's objective is to extract the highest rent from its monopoly position. Their objectives are conflicting because higher monopoly prices are associated to lower consumption and higher goods prices,that's to say lower welfare in the CIC. At the same time, changes in monetary policy influence aggregate demand and, as a result, the CEC's profits through the extensive margin. The optimal actions of both players depend on each other and also on the production decisions of the fringe. For example, an excessively high level of ψ will induce the central bank to rise rates, thus eventually reducing profits. General equilibrium effects will matter as well. Both policies have an impact on the price of the commodity which in turn defines the number of fringe producers that enter the market and, as a result, the market power of the CEC.
In our model both players act strategically, taking into account the other player's reaction and general equilibrium constraints, and most importantly the supply of fringe producers.
Formally, the exporter's game is defined as:

$$\begin{aligned}
&\max_{\Gamma} E_t(\Pi_t) \mid \Phi^* \\
&\text{s.t. } E_t\Omega(x_{t-1}, x_t, x_{t+1}, \eta_t) = 0 \\
&\psi_t = \psi_{ss}\left(\frac{P_{Q,t}}{P_{Q,t-1}}\right)^{\psi_Q}\left(\frac{Y_t}{Y_{t-1}}\right)^{\psi_Y},
\end{aligned} \tag{3.11}$$

where $\Phi^* = \{\phi^*_{head}, \phi^*_{core}, \phi^*_{com}, \phi^*_Y\}$ is the optimal strategy of the central bank conditional on the strategy of the CEC, i.e. $\Gamma = \{\psi_{ss}, \psi_Q, \psi_Y\}$. CEC's strategy involves the choice of three parameters: the steady state mark-up, ψ_{ss}, the sensitivity of the mark-up to output, ψ_Y, and commodity price (ψ_Q) fluctuations. We allow the exporter to both set the *deterministic* steady state of the commodity price above the efficient allocation in this model, i.e. $\psi_{ss} = 1$, and to react to the business cycle volatility. Given this, the exporter can also directly change the volatility component of P_Q in the *stochastic* steady state of the model. Specifically, she might increase (decrease) the mark-up in periods of high (low) demand -to reap the benefits of less elastic commodity consumption in the CIC- or change the mark-up when commodity prices rise (fall) to keep out the fringe production or sustain aggregate demand for the commodity at a global level -thus preserving its intensive margin. To discipline the model's outcome, we constrain these parameters as follows: $\psi_{ss} \geq 0.4$, $\psi_Q \in [-2, 2]$, $\psi_Y \in [-2, 2]$.
The CIC's central bank chooses the parameters of Equation (3.5) by solving:

$$\begin{aligned} &\max_{\Phi} E_t(W_t) \mid \Gamma^* \\ &\text{s.t. } E_t\Omega(x_{t-1}, x_t, x_{t+1}, \eta_t) = 0. \end{aligned} \tag{3.12}$$

where the strategy vector is $\Phi = \{\phi_{head}, \phi_{core}, \phi_{com}, \phi_Y\}$. Moreover we let the central bank choose the price variable to target, specifically whether to respond to core or headline inflation. This choice has strategic implications because headline inflation reacts immediately to fluctuations in commodity prices, proportionally to the share of commodities in consumption. It also introduces a source of non-linearity in the game because the reaction function of the central bank has kinks around $\phi_{head} = 0$ or $\phi_{core} = 0$. We also allow the central bank to directly react to commodity prices. Filardo et al. (2020) show that, absent strategic interactions, it is not optimal to directly react to commodity price fluctuations, i.e. $\phi_{com} = 0$. However, that might change when accounting for strategic interactions. One strategy for the central bank could be to *credibly threaten* to rise the policy rate in response to stronger commodity prices to impose discipline, i.e. limit the monopoly power, on the dominant commodity exporter. However, the threat might not be credible in general equilibrium because of time inconsistency, i.e. when commodity prices actually rise, the central bank might have the incentives to lower policy rates instead of enforcing its threat. The central bank then faces a trade-off between a more accommodative monetary policy stance, entailing higher welfare and commodity prices, and a tighter monetary policy, with lower welfare is lower and reduced mark-ups on the part of the CEC. Such trade-off is not linear because the price of commodities has second- and third-round effects on welfare, as commodities are both consumed by households and used in production. When defining the strategies of the central bank, we additionally impose that the Taylor principle holds and constrain the parameters of the policy rule, in line with standard practice in the DSGE literature: $\phi_{core} \in [0, 2.5]$, $\phi_{head} \in [0, 2.5]$, $\phi_{com} \in [0, 2]$, $\phi_y \in [0, 2]$.[22]

Finally we consider two calibrations of the model, one where commodities account for a small share of production and consumption (i.e. $\alpha = \gamma = 0.1$) and a second in which commodities are more relevant ($\alpha = \gamma = 0.3$). In this way we can study how the Nash equilibrium changes when the importance of commodities in the economy increases. We name these two calibrations "commodity independent" and "commodity intensive" respectively.

We use `DSGE Nash` to compute both the solution of the Nash game between the two countries and the optimal parameters when each problem is solved independently, i.e. by setting the parameters of the other country as in Table 2 and excluding strategic interactions. The outcome variables, welfare and profits, are computed based on a second-order simulation of the model with pruning, see Born and Pfeifer (2020).

Table 3: Optimal parameter under individual policies and Nash game, commodity independent economy ($\alpha = \gamma = 0.1$)

Variable	Baseline (1)	Optimal Markup (2)	Optimal Mon. policy (3)	Nash game (4)
ϕ_{head}	0.00	0.00	0.00	0.00
ϕ_{core}	1.50	1.50	2.40	2.40
ϕ_{com}	0.00	0.00	0.00	0.00
ϕ_Y	0.00	0.00	0.00	0.00
ψ_{ss}	1.00	0.90	1.00	0.90
ψ_Q	0.00	0.00	0.00	0.00
ψ_Y	0.00	-0.10	0.00	-0.10
$E(\mathcal{W})$	-122.20	-121.35	-122.12	-121.27
$\xi \times 100$	0.00	0.85	0.08	0.93
$E(\Pi) \times 100$	4.43	4.44	4.45	4.46
$E(C)$	0.55	0.56	0.55	0.56
$E(P_Q)$	2.26	2.18	2.26	2.18
$E(R) \times 4$	3.84	3.84	3.96	3.96

Notes: optimal policy parameters from the individual optimization of the commodity exporter (column 2), the commodity importer (column 3) and the solution of the Nash game (column 4). Welfare and profits are computed based on a second-order solution of the model with pruning. In this calibration of the model, commodities account for 10% of production and consumption in the steady state. Interest rates are annualized. The consumption equivalent is computed relative to the baseline calibration of the model as $\xi = \exp\left[(1-\beta)\, E\left(\mathcal{W}^{policy} - \mathcal{W}^{baseline}\right)\right] - 1$.

When the share of commodity in consumption and production is relatively small, both the commodity exporter and the commodity importer have a dominant strategy: the central bank implements classic inflation targeting ($\phi_{core} = 2.4$), while the commodity exporter applies a mild mark-down to the commodity price ($\psi_{ss} < 1$) and weakly reacts to changes in output.

[22] The constrains on the Taylor rule parameters ($\phi_{core} + \phi_{head} + \phi_{com} + \phi_y > 1$ and $\phi_{core} > 0 \,|\, \phi_{head} > 0$) are imposed using the `exclusion_condition` option. See Appendix C for a description of this option.

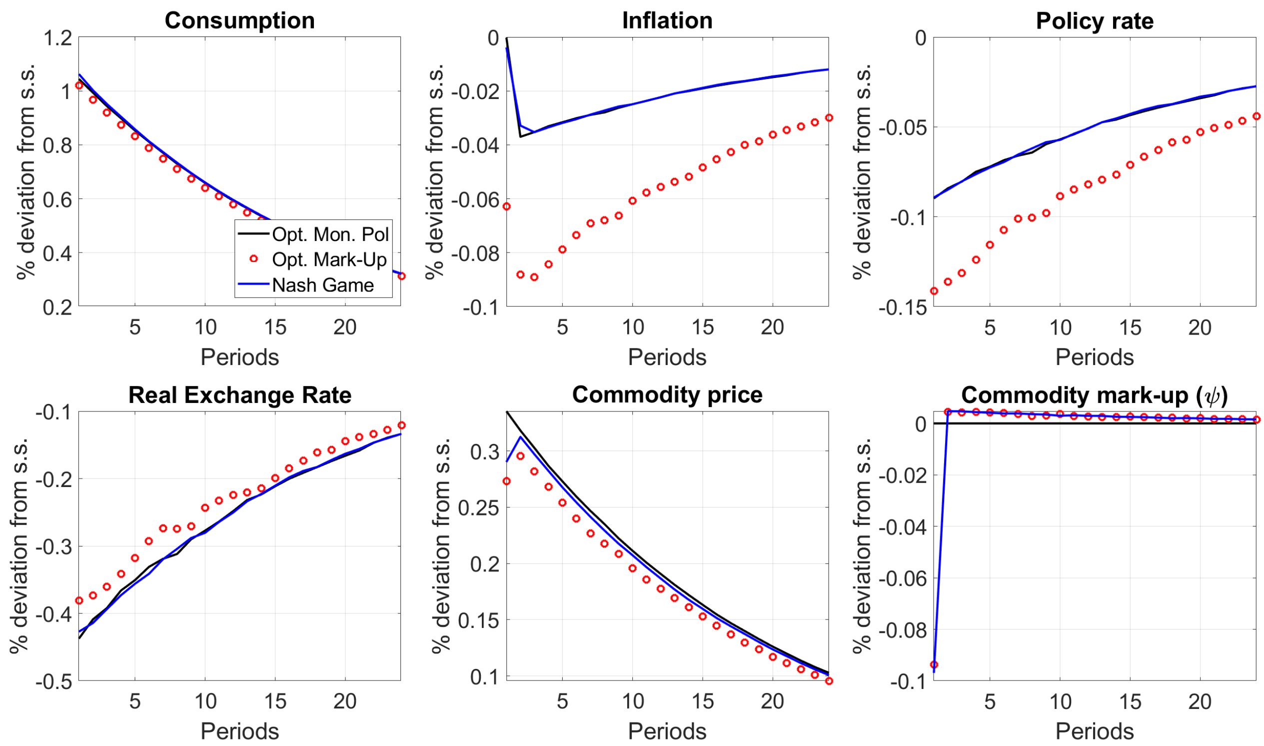
Since this is a dominant strategy, it is optimal not only under the Nash equilibrium, (blue line in Figure 1), but also when policies are decided independently (black line and red dots in Figure 1), see columns (2)-(4) of Table 3. In particular, the dominant commodity exporter finds optimal to slightly cut the price of the commodity to keep the fringe out of the market as much as possible. By setting a low ψ_{ss}, indeed, the CEC prevents the fringe from reducing the leader's market power and, in the equilibrium, obtains higher profits than in the alternative scenario of an higher ψ_{ss}. In other terms, the lower commodity prices are compensated by the higher share of commodity supplied by the dominant exporter. If ψ_{ss} was instead higher, more fringe competitors could engage in production and they would eventually erode profits on the intensive (quantities) margin, see $E(\Pi)$ in Table 3. Such competition eventually benefits the commodity importer. Under the optimal policy for the CEC, in fact, welfare in the importing country increases. This happens because commodity prices are lower and that has positive effects on consumption and production. Optimal monetary policy also increases welfare through a tighter reaction to inflation. It is also worth noticing that, when commodities are relatively less important in the domestic economy, the central bank finds it optimal to follow a standard inflation targeting rule. Notably, optimal policy implies a somewhat tighter reaction to inflation, with a rise in policy rates by 12 basis points (bps) on average, see Table 3. However, gains in terms of welfare are mainly driven by the competition in commodity production. Comparing column (2) and column (4) in Table 3, indeed, highlights that only 0.08 out of 0.93 gains in consumption equivalents are due to monetary policy. These results depend on the calibration of α and γ. When commodity prices are relative less important for inflation and output, commodity demand is relatively low. That in turn implies low incentives for fringe producers and a lower sensitivity of the commodity price to demand changes. In this environment, the CEC finds it optimal to reduce the mark-up relatively more to substantially increase its commodity supply. Nonetheless, welfare improves because the commodity is 4% cheaper while consumption[23] is 2% higher. At the same time, given the limited role played by the commodities in the economy, the central bank can afford to "look though" commodity prices and focus on domestic variables (inflation), see Blanchard and Galí (2007) and Blanchard and Galí (2007).

Figure 1 reports the impulse response functions, at second order, for a total factor productivity shock under the three calibrations of Table 3. As the equilibrium is in dominant strategies, impulse responses for domestic variables under optimal monetary policy and under the Nash equilibrium are relatively similar, i.e. each agent would choose the same strategy independently of the actions of the others. This also reflects the relatively lower weight of the commodity

[23]Recall that the commodity is part of the consumption aggregator.

in consumption and production, i.e. commodity price developments have reduced impact on domestic variables, because commodity demand is limited. Notably, under the optimal mark-up policy -and with monetary policy at its baseline calibration- the commodity price reacts by about 10% less to a TFP shock. In this scenario, monetary policy -which is not optimal- is less effective in smoothing business cycle fluctuations, which results in somewhat lower output and consumption after the shock. From the standpoint of the dominant exporter, this reduces demand for the commodity, whose price falls in spite of a negative ψ_Y.

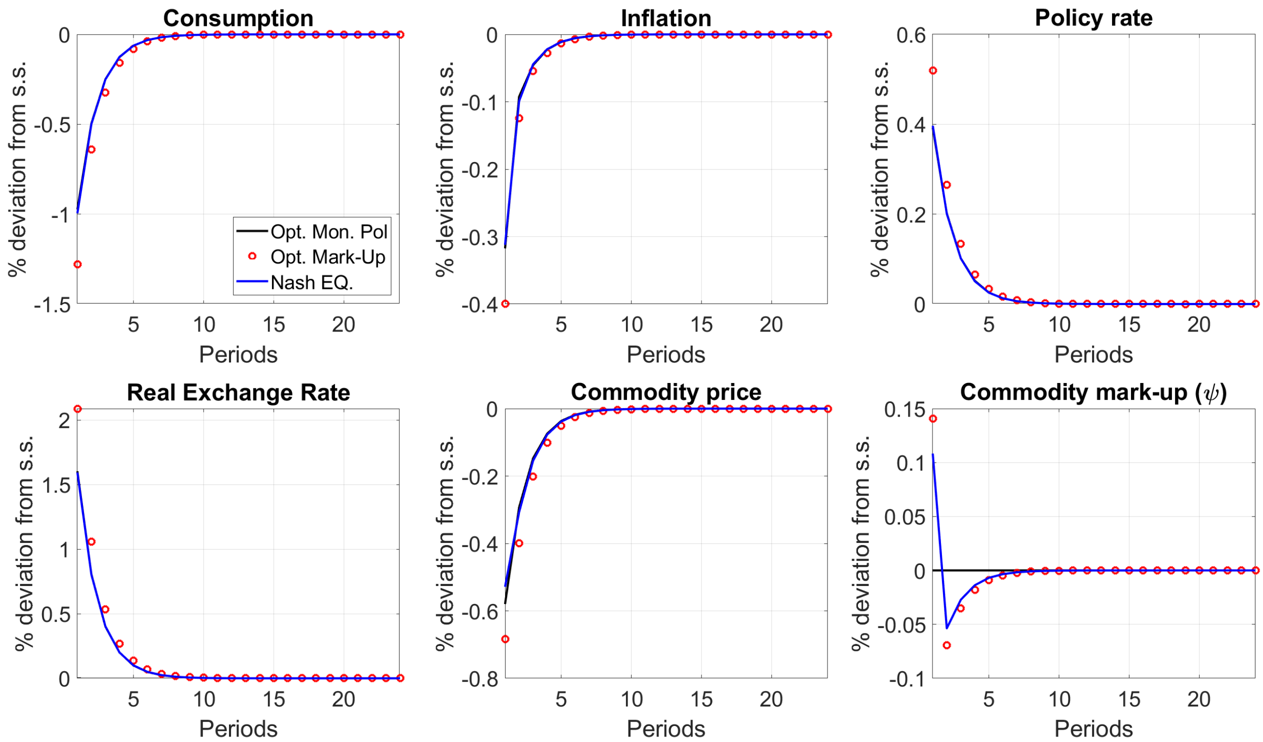
Figure 1: Impulse responses to a positive TFP shock, commodity independent economy



Notes: impulse responses for the parametrization reported in Table 3. IRFs are computed by solving the model at second order with pruning.

Figure 2 reports impulse responses to a monetary policy shock in the importing economy. Differences across policy regimes are similar to those discussed for a TFP shock. Impulse responses profiles under optimal monetary policy and the Nash equilibrium are similar for the CIC because the central bank has a dominant strategy. As regards monetary policy shocks, relevant discrepancies emerge when the central bank does not behave optimally. In this case, the monetary authority tolerates higher inflation rates, which entails an amplification of the monetary policy impact on prices. This has consequences for the dominant exporter that instead behaves strategically. Under this scenario, contractionary (expansionary) monetary policy shocks generate larger fluctuations in domestic variables, which in turn decrease (increase) the commodity demand to a larger extent. As a result, the dominant exporter is forced to raise (drop) more the commodity price -in the order of 30%- so to keep the fringe out of the market. Notice also that consumption, and consequently output, are more volatile soon after the initial shock when the monetary policy rule is not optimized. For this reason, they bounce faster back to equilibrium after the initial, larger, fall. This also explains why the commodity exporter is forced to first increase and then decrease ψ (bottom-left panel of Figure 2). Figure A.1 and Figure A.2 in the Appendix A report impulse responses for a preference shock and a commodity price shock respectively. Results are broadly similar to those discussed for the previous shocks. The more marked differences are due to the limited stabilization of the economy through monetary policy after a preference shock, which in this model affects the inter-temporal discount factor, i.e. the choice between consumption and saving.

Figure 2: Impulse responses to a tightening monetary policy shock, commodity independent economy



Notes: impulse responses for the parametrization reported in Table 3. IRFs are computed by solving the model at second order with pruning.

We now consider the case of an economy that relies more on commodities, i.e. where the commodity's weight in production and consumption is around 30%. This scenario (Table 4) is strikingly different from the case of a commodity independent economy. First, and most importantly, the central bank does not have a dominant strategy. In other terms, due to the high weight of commodities in the economy, it is forced to adopt a sub-optimal monetary policy strategy which implies a lower-than-desired sensitivity to the commodity price. In other terms, absent the reaction of the commodity exporter, the central bank would opt for a more hawkish stance. Second, as the commodity is now an important component of production and consumption baskets, it becomes optimal to target *headline* rather than core inflation. Third, the commodity exporter plays the same strategy, with a low steady-state mark-up ($\psi_{ss} = 0.5$) and a strong reaction to output and the commodity price ($\psi_Q = \psi_Y = -0.5$) under both the optimal mark-up and the Nash equilibrium. In the optimal mark-up case (column (2)) the stronger demand in the importing country generates price pressures in the commodity market. As the price goes up, more fringe producers enter the market, thus eroding the market power of the CEC. Anticipating this, the dominant exporter seeks to block the entrance of fringe producers by reducing its mark-up upfront. Along the same reasoning, the CEC also adjusts prices more aggressively. Since demand is high, the lower mark-up has limited effects on profits, that remain higher than in the previous scenario. This happens because the price reduction (10%) is more than compensated by significantly larger market shares for the dominant exporter. As a result, profits increase by 4%.

In the Nash equilibrium, the central bank has to deviate from its optimal strategy. In particular, the reaction to commodity prices is 60% lower compared to optimal monetary policy (columns (3) and (4) in Table 4). A strong reaction to the commodity price, indeed, would reduce domestic demand, as higher rates compress demand for all goods, including the commodity, and limit the entrance of new commodity producers. This strategy, with higher ϕ_{com} as in column (3), benefits the commodity producer that could increase the mark-up by exploiting the preemptive effect of the central bank's policies on the fringe. In other terms, the central bank would contribute to constraint the entrance into the commodity market to the benefit of the dominant producer, who could eventually extract a higher rent. This scenario, however, would reduce significantly welfare in the domestic economy.[24] For this reason, the central bank internalizes the reaction of the monopoly exporter and lowers its sensitivity to commodity prices, i.e. it tolerates higher commodity price inflation. By doing this, the dominant exporter cannot rise the mark-up because the higher price, not compensated by a stronger policy response, would attract more new entrants and reduce its market power and production share. These dynamics are an example of strategic interactions. The central bank knows that its opponent would take advantage of an excessively hawkish policy stance. It then prevents that scenario by trading a less efficient monetary policy, which implies somewhat stronger price fluctuations and some degree of welfare losses, against a low commodity mark-up, which generates welfare

[24] This does not hold under the optimal mark-up policy scenario because the central bank's reaction is still based on calibrated parameters (column (3) of Table 4). In that case, the central bank targets core inflation, which reacts significantly less to commodity prices. It follows that the central bank's choices have less impact on commodity markets and on the entrance of new fringe producers.

gains. Similarly, the dominant exporter knows that if she increases the mark-up under the Nash equilibrium, the less aggressive policy stance would induce more fringe producers to enter the market, thus eroding her profits. For this reason, she optimally chooses to set the mark-up to a very low level. Under the Nash equilibrium, welfare is significantly higher than in the optimal monetary policy case, by about 17.64 in consumption equivalent terms. This is due to the fact that the central bank is able to contain the rise in commodity prices.

Table 4: Optimal parameter under individual policies and Nash game, commodity dependent economy ($\alpha = \gamma = 0.3$)

Variable	Baseline (1)	Optimal Markup (2)	Optimal Mon. policy (3)	Nash game (4)
ϕ_{head}	0.00	0.00	2.40	2.40
ϕ_{core}	1.50	1.50	0.00	0.00
ϕ_{com}	0.00	0.00	2.00	1.20
ϕ_Y	0.00	0.00	0.00	0.00
ψ_{ss}	1.00	0.50	1.00	0.50
ψ_Q	0.00	-0.50	0.00	-0.50
ψ_Y	0.00	-0.50	0.00	-0.50
$E(\mathcal{W})$	-292.33	-276.09	-292.21	-275.99
$\xi \times 100$	0.00	17.63	0.11	17.75
$E(\Pi)\times 100$	8.98	9.37	8.99	9.37
$E(C)$	0.17	0.19	0.17	0.19
$E(P_Q)$	3.18	2.85	3.18	2.85
$E(R)\times 4$	3.80	4.00	3.80	4.00

Notes: optimal policy parameters from the individual optimization of the commodity exporter (column 2), the commodity importer (column 3) and the solution of the Nash game (column 4). Welfare and profits are computed based on a second-order solution of the model with pruning. In this calibration of the model, commodities account for 30% of production and consumption in the steady state. Interest rates are annualized. The consumption equivalent is computed relative to the baseline calibration of the model as $\xi = \exp\left[(1-\beta)\, E\left(\mathcal{W}^{policy} - \mathcal{W}^{baseline}\right)\right] - 1$.

Impulse responses in this scenario are also markedly different. Consider a positive TFP shock in the domestic economy. When the dominant commodity exporter optimally sets the policy rule (red dots in Figure 3) the business cycle in the domestic economy is significantly more volatile. Although inflation is higher, the central bank eases the policy rate. This happens because in the baseline calibration the central bank targets core inflation but the commodity price has a strong weight in the production and consumption baskets. For similar reasons, the commodity price increases, because more commodity is demanded, in particular by firms. The CEC reacts by lowering the mark-up, to keep fringe competitors out of the market, but this action does not limit the commodity price increase, that rises about 3 times more than in the Nash equilibrium. This sharp surge leads to higher inflation, despite the lower cost of production. When the central bank, instead, acts optimally and keeps the policy rule to its baseline calibration, shocks have lower effects on the domestic business cycle and inflation drops after a positive TFP shock, in line with standard theory. This is due to the more limited demand for the commodity, which contains the price increase, especially on impact (Figure 3).

When both players act strategically, the central bank is able to get very close to the optimal policy allocation by *reducing* its sensitivity to price fluctuations. Notably, the central bank reacts by around 60% less to commodity price changes. In this equilibrium, the monetary authority is forced to accept higher inflation to prevent a rise in the mark-up, which would reduce households' welfare. Specifically, should the central bank sterilize commodity price changes, demand would shrink and fringe producers would be kept out of the market. If the role of fringe producers was limited, the dominant exporter could rise the mark-up and extract a higher rent, because competitors would have relatively weaker incentives to enter. This rent is a cost for consumers and firms in the importing economy and, hence, reduces welfare. All in all, under strategic interactions, if the monetary authority sets the policy rule as in column (3) of Table 4, the exporter will deviate from its original strategy by increasing the mark-up. That being said, policy makers in the CIC anticipate this strategic behavior and lower ϕ_{com} in advance. As a consequence, the dominant exporter finds it optimal to keep the mark-up low so to prevent more fringe producers from entering the market.

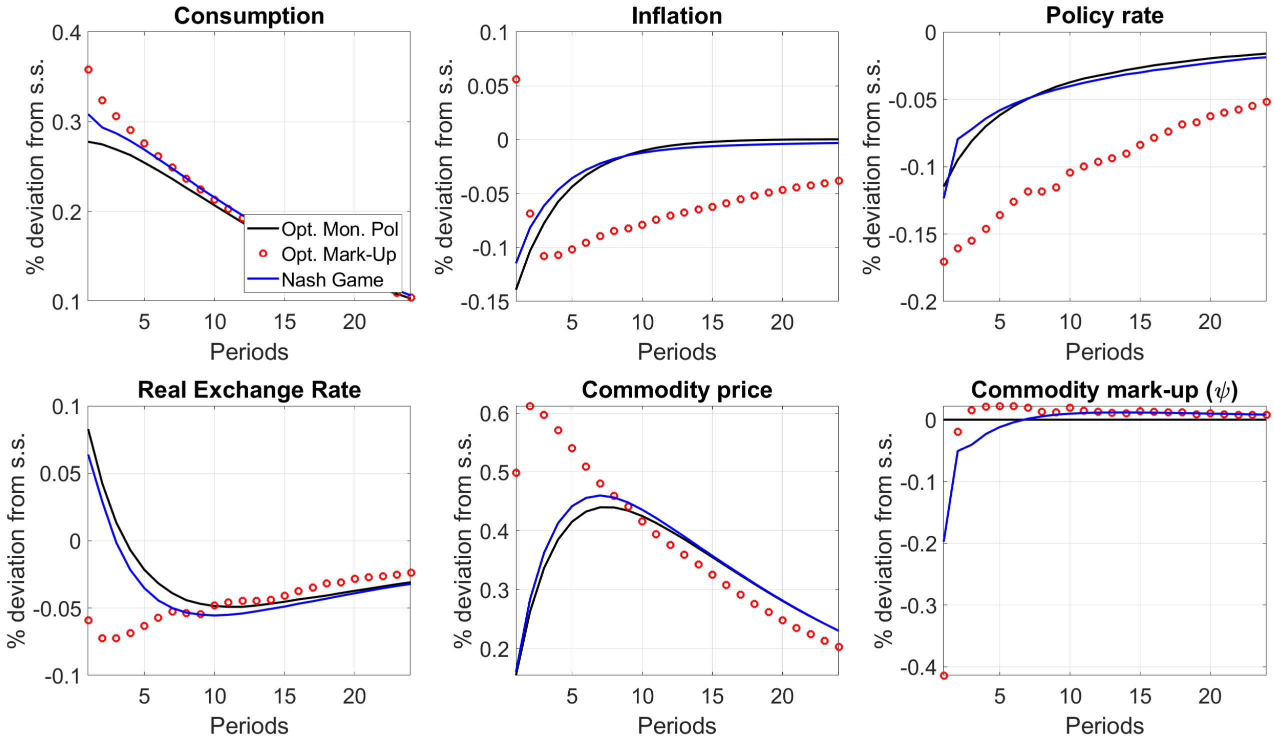
A similar logic applies when considering a monetary policy shock (Figure 4). In this case business cycle volatility is higher under the optimal mark-up policy and the commodity price fluctuates more. By tolerating stronger commodity price fluctuations the central bank is able to reduce the market power of the dominant exporter and impulse responses are close to the optimal monetary policy case. Figure A.3 and Figure A.4 in the Appendix A reports the responses to preference and commodity price shocks.

4 Conclusion

This paper presents `DSGE Nash`, a toolkit designed to solve global games and optimal policy problems in macroeconomic models. When considering strategic interactions, i.e. when agents engage in a "game", the resulting equilibria are generally different from those attained under cooperation. This in turn has relevant implications for policy evaluation and normative suggestions.

We propose an algorithm based on numerical methods that detects pure strategy Nash equilibria in policy games involving N players and strategies. In other terms, it solves a global game across a model's agents. Besides some practical advantages (e.g. speed, flexibility in the choice of objective function), the toolkit envisages four main configurations that allow its application to a wide range of problems. First, it can be used to solve a Nash game involving some of the

Figure 3: Impulse responses to a positive TFP shock, commodity dependent economy

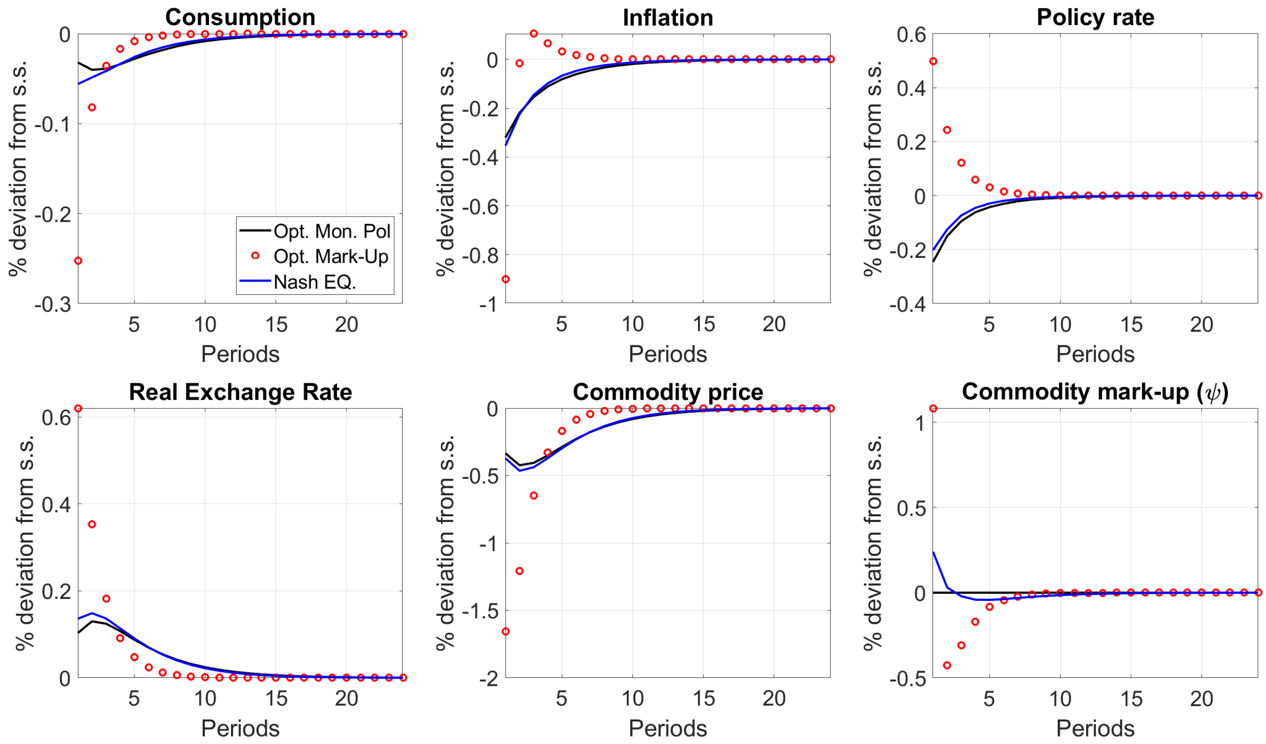


Notes: impulse responses for the parametrization reported in Table 3. IRFs are computed by solving the model at second order with pruning.

agents of the model, where the remaining structural equations are taken as constraints. Second, it can target a wide range of variables, policy functions (i.e. Taylor rules, tax rules, capital constraints...) and be applied to different classes of macro models. Third, when only one player is selected, `DSGE Nash` re-casts the problem into a standard optimal policy problem and solves it. Fourth, it can estimate models by moment or by impulse response matching. These last two applications leverage on the similarity between the algorithms used to solve the Nash game and to estimate the model. All these functionalities are provided in an user-friendly environment and allow for a great deal of customization of the model or the solution algorithm.

We provide a practical example of how to use `DSGE Nash` in the context of an open-economy model where a commodity importing country and a monopolist compete on the commodities market, that is characterized by barriers to the entrance for new producers. We find that the degree of strategic competition between the two agents depends on the importance of the commodity in consumption and production. If the commodity accounts for a low share of these baskets, both players have a dominant strategy that implies strong core inflation targeting and no reaction to the commodity price for the commodity importer, as in Blanchard and Galí (2007) and Filardo et al. (2020), and a somewhat lower mark-up setting for the monopolist. Since the commodity price has a reduced impact on the importing economy, the interactions between the

Figure 4: Impulse responses to a tightening monetary policy shock, commodity dependent economy



Notes: impulse responses for the parametrization reported in Table 3. IRFs are computed by solving the model at second order with pruning.

two players are limited and relatively simple. The problem becomes more complicated when the commodity is important in the importer's economy. In that case, the central bank cannot follow the "first-best" policy because that would allow the exporter to extract higher rents with detrimental effects on welfare in the commodity-importing economy. The central banks anticipates this scenario and chooses an alternative rule that tolerates higher commodity prices. Because of that, it gets easier to enter the commodity market and the exporter faces more competition. To avoid having new competitors, the exporter keeps the commodity price low, which benefits the importing economy and ultimately increases welfare, despite a less efficient monetary policy rule. This example highlights the importance of strategic interactions. The key element that allows for higher welfare is not the efficiency of the policy rule chosen, but rather the "discipline" imposed onto the commodity exporter, which limits her willingness to increase the rent.

In conclusion, `DSGE Nash` provides an handy tool to address these types of policy questions also in frameworks other than DSGE models. Users can indeed feed the algorithm with any custom-made policy function. The code will then compute payoffs for each policy parameter and evaluate the outcomes for each players under different strategies. Moreover, the same set of algorithms used to compute the Nash equilibrium and optimal policies can be applied to the calibration of models by matching empirical data. These features will hopefully lower the barriers to entry to the computational analysis of macro models. Further developments could include the extension to mixed-strategy equilibria.

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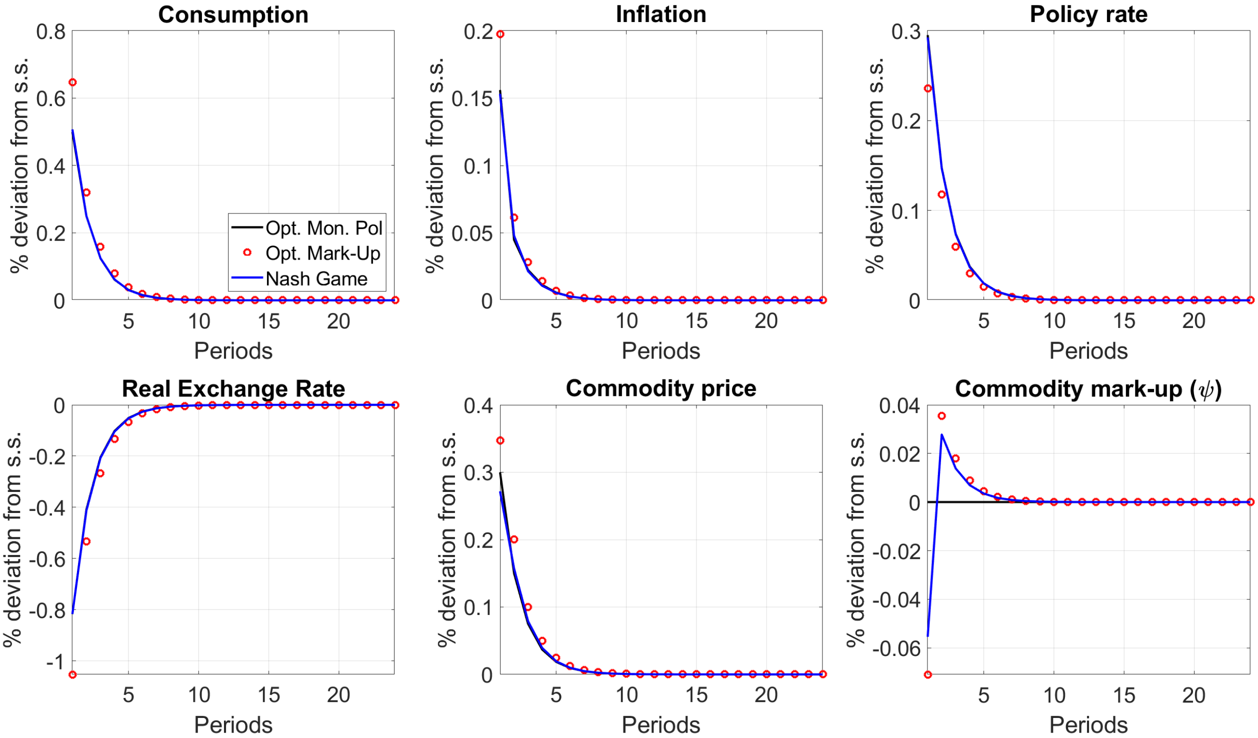
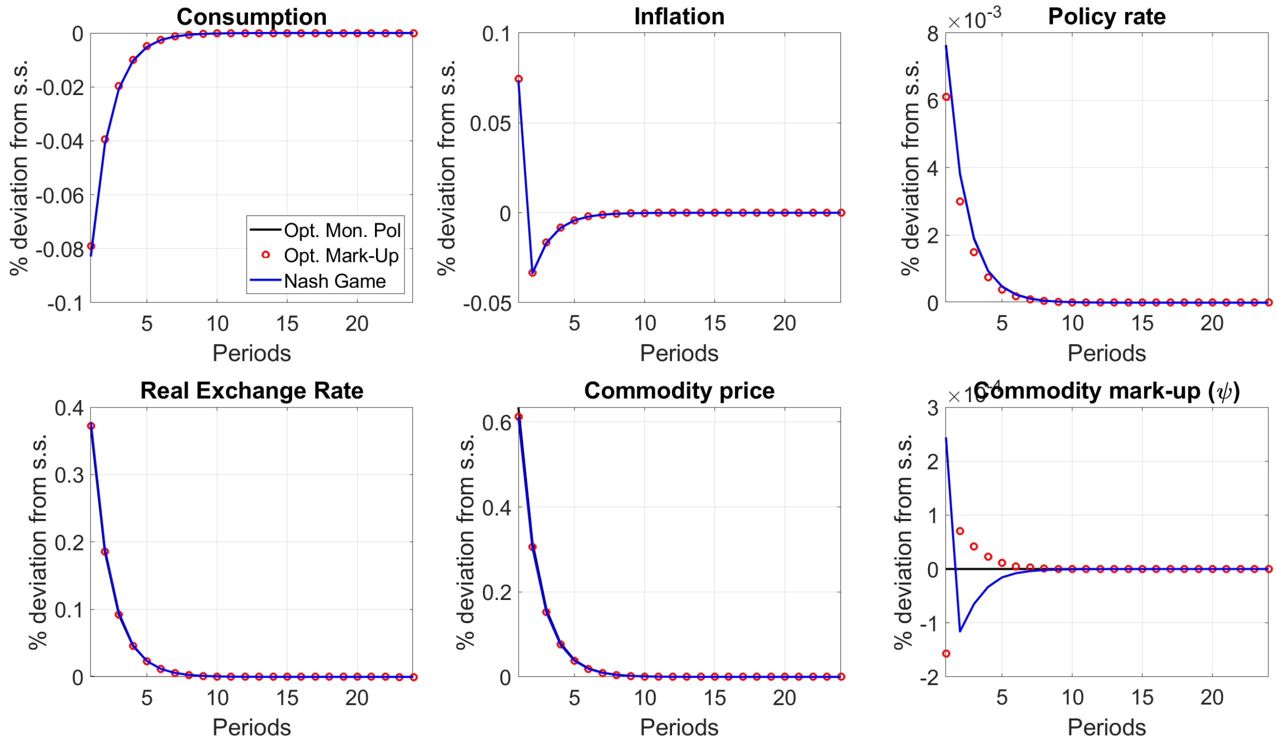
A Figures

Figure A.1: Impulse responses to a positive preference shock, commodity independent economy



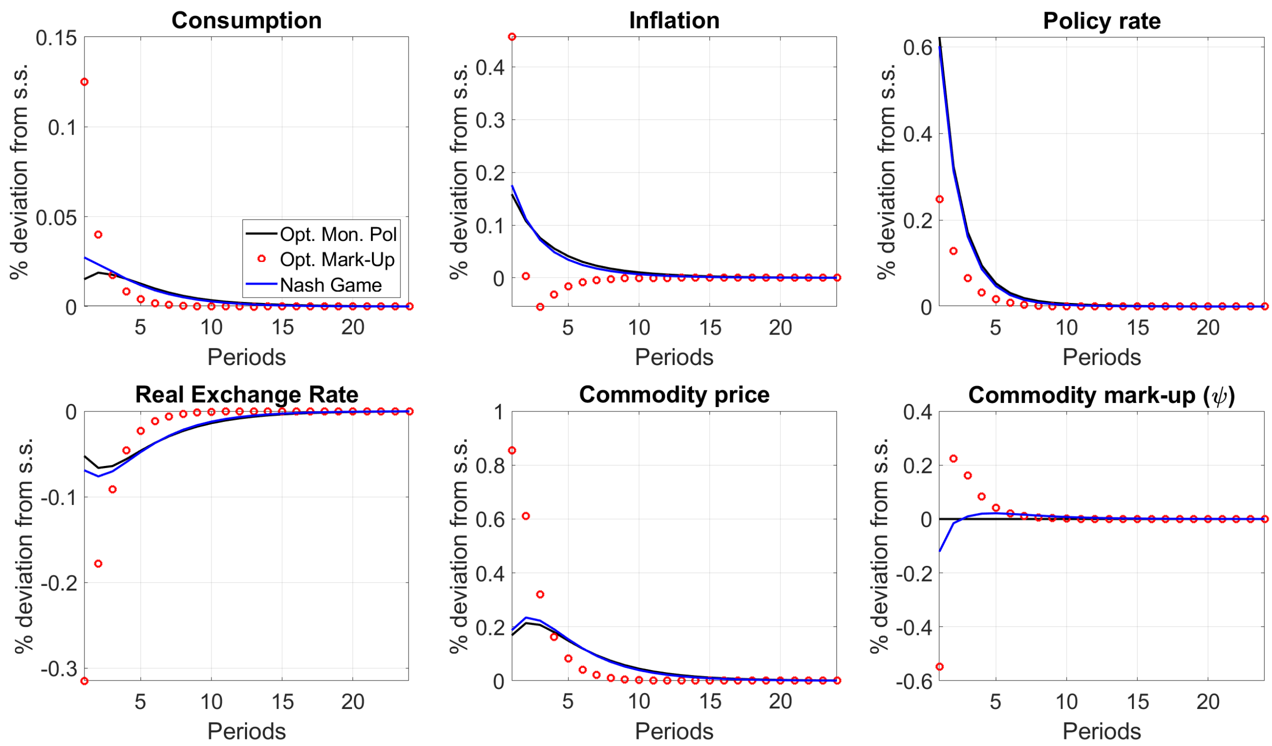
Notes: impulse responses for the parametrization reported in Table 3. IRFs are computed solving the model at second order with pruning.

Figure A.2: Impulse responses to a positive commodity price shock, commodity independent economy



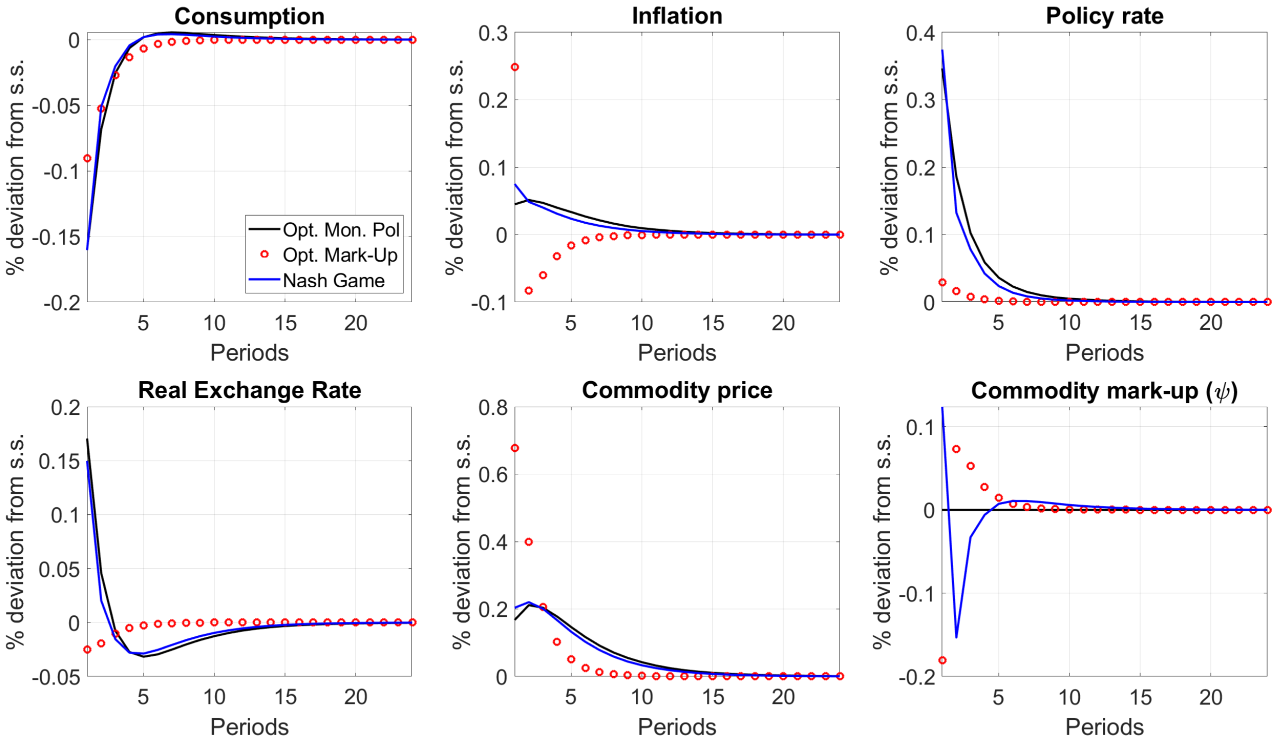
Notes: impulse responses for the parametrization reported in Table 3. IRFs are computed solving the model at second order with pruning.

Figure A.3: Impulse responses to a positive preference shock, commodity dependent economy



Notes: impulse responses for the parametrization reported in Table 3. IRFs are computed solving the model at second order with pruning.

Figure A.4: Impulse responses to a positive commodity price shock, commodity dependent economy



Notes: impulse responses for the parametrization reported in Table 3. IRFs are computed solving the model at second order with pruning.

B The model

The model closely follows Nakov and Pescatori (2010) and Filardo et al. (2020). In the commodity-importing economy, consumers maximize the stream of discounted expected utility. The period utility function depends on consumption (C), labor (L) and an exogenous preference shock (e^C):

$$U_t = \exp e_t^C \ln(C_t) - \frac{L_t^{1+\nu}}{\nu}, \tag{B.1}$$

with aggregate consumption combining consumption of goods (C_Y) and commodities ($\mathcal{M}_Q$): $C_t = (C_{Y,t})^{1-\gamma} (\mathcal{M}_{Q,t})^{\gamma}$. Households are subject to a budget constraint. Sources of funds are returns on risk-free bonds (B) held between period $t-1$ and t, the wage bill (WL) and revenues from firms (Γ). Uses of funds are taxes (T), consumption and purchase of new safe assets. The representative household's budget constraint is:

$$C_t = P_{Q,t}\mathcal{M}_{Q,t} + P_{Y,t}C_{Y,t} = W_t L_t + R_t B_{t-1} - B_t + \Gamma_t - T_t \tag{B.2}$$

First order conditions (where $w_t = \frac{W_t}{P_t}$ is the real wage) are:

$$\begin{aligned} 1 &= \beta \frac{R_t}{\pi_{t+1}} \frac{\exp\left(e_{t+1}^C\right)}{\exp\left(e_t^C\right)} \frac{C_t}{C_{t+1}} \\ w_t &= C_t L_t^{\nu} \\ C_{Y,t} &= (1-\gamma)\frac{P_t}{P_{Y,t}} C_t \\ \mathcal{M}_{Q,t} &= \gamma \frac{P_t}{P_{Q,t}} C_t \end{aligned} \tag{B.3}$$

with the Lagrangian multiplier associated to the problem being $\lambda_t = \frac{1}{C_t}$. Substituting the first order conditions into the consumption aggregator allows to define the aggregate price index (P) as:

$$P_t = (P_{Y,t})^{1-\gamma} (P_{Q,t})^{\gamma} \tag{B.4}$$

Perfect competitive firms produce intermediate output by combining labor and commodity ($\mathcal{M}_Y$). They choose the share of commodity and labor in production to maximize profits subject the production function $Y_t = \exp(A_t)(L_t)^{1-\alpha}(\mathcal{M}_{Y,t})^{\alpha}$, where A is a total factor productivity shock. Period-profits are given by:

$$\Pi_t^f = P_t^f Y_t - W_t L_t - \mathcal{M}_{Y,t} P_{Q,t} \tag{B.5}$$

First order conditions imply:

$$
\begin{aligned}
MC_t &= \frac{W_t^{1-\alpha} Q_t^{\alpha}}{\exp\left(A_t\right)(1-\alpha)^{1-\alpha}\alpha^{\alpha}} \\
L_t &= (1-\alpha)\frac{MC_t}{w_t} Y_t \\
\mathcal{M}_{Y,t} &= \alpha \frac{MC_t}{P_{Q,t}} Y_t
\end{aligned}
\tag{B.6}
$$

where MC is the real marginal cost. Because of perfect competition $P^f = MC$.

Intermediates goods are sold to retailers which bundle them and sell aggregate final goods to consumers (in both countries). Retailers hold some degree of market power and hence set final prices above the marginal cost. They can however reset prices only with probability $1-\theta$. Their objective function is:

$$
E_t \sum_{k=0}^{\infty} \theta^k \beta^k \frac{C_t}{C_{t+k}} \frac{P_{Y,t}}{P_{Y,t+k}} \left[1 - \tau P_{Y,t} - P_t MC_t\right] \left(\frac{P_{Y,t}}{P_t}\right)^{-\epsilon} Y_t \tag{B.7}
$$

where ϵ is the elasticity of substitution of consumption, τ a steady-state tax and $\beta^k \frac{C_t}{C_{t+k}}$ the stochastic discount factor. First order conditions determine price setting. They can be written in recursive form as:

$$
\begin{aligned}
\mathcal{D}_t &= \frac{Y_t}{C_t} + \theta\beta E_t\left(\pi_{core,t+1}^{\epsilon-1}\mathcal{D}_{t+1}\right) \\
\mathcal{N}_t &= \frac{Y_t}{C_t}\frac{\epsilon}{\epsilon-1} MC_t \frac{Y_t}{C_t} + \theta\beta E_t\left(\pi_{core,t+1}^{\epsilon-1}\mathcal{N}_{t+1}\right) \\
\theta\pi_{core,t+1}^{\epsilon-1} &= 1 - (1-\theta)\left(\frac{\mathcal{N}_t}{\mathcal{D}_t}\right)^{1-\epsilon} \\
\Delta_t &= (1-\theta)\left(\frac{\mathcal{N}_t}{\mathcal{D}_t}\right)^{\epsilon} + \theta\Delta_{t-1}\pi_{core,t}^{\epsilon}
\end{aligned}
\tag{B.8}
$$

Each fringe producer i, instead, produces with the technology:

$$
\mathcal{M}_t^{cf}(i) = Z_t \omega(i) \tag{B.9}
$$

where $\omega(i)$ is an idiosyncratic component and $\frac{1}{\omega(i)}$ is assumed to have a uniform distribution, $F\left(\frac{1}{\omega(i)}\right)$, in the interval from a to b. Profits (recall that prices are globally set in the currency of the commodity-importing economy) for each fringe producer are:

$$
P_{Q,t}\mathcal{M}_t^{cf}(i) - \frac{P_{Y,t}}{P_t}\frac{\mathcal{M}_t^{cf}(i)}{\omega(i) Z_t} \tag{B.10}
$$

Assuming that the total mass of competitive fringe countries is Ω_t, the aggregate amount of the commodity produced by the competitive fringe is given by:

$$\mathcal{M}_t^{cf} = \int_0^{\Omega_t} \mathcal{M}_t^{cf}(i)d(i) = \Omega_t F\left(P_{Q,t} Z_t\right) \tag{B.11}$$

which can be further simplified into $\mathcal{M}_t^{cf} = P_{Q,t} Z_t \Omega_t$. Moreover, the fringe mass follows an AR(1) process: $\Omega_t = \rho_\Omega \Omega_{t-1} + \varepsilon_t^\Omega$. The utility function of households in the dominant commodity exporting country is:

$$E_t \sum_{t=0}^{\infty} \beta^t \ln C_t^{cec}. \tag{B.12}$$

We assume that households' revenues in the CEC are equal to profits from commodity trade. In other terms their budget constraint is:

$$Pi_t = \mathcal{M}_t^{cec}\left(\Psi_t P_{Q,t} - Z_t^{-1}\right) = P_t^C C_t^{cec}. \tag{B.13}$$

The dominant exporter maximises Equation (B.12) subject to Equation (B.13), the global commodity demand ($\mathcal{M}_{Y,t} + \mathcal{M}_{Q,t} = \mathcal{M}_t^{cec} + \mathcal{M}_t^{cf}$) and Equation (B.11) (i.e., by internalizing the supply of the fringe). The resulting first order condition is:

$$P_{Q,t} = \Psi_t Z_t^{-1} \tag{B.14}$$

with $\Psi_t = \psi_t \left(1 + \frac{\mathcal{M}_t^{cec}}{2\mathcal{M}_t^{cf}}\right)$.

C User Manual

C.1 Presentation

This note presents `DSGE Nash`, a toolkit, written in Matlab, to compute the Nash equilibrium, in pure strategies, of strategic games in macro models. Strategies are entered in grids which `DSGE Nash` loops over to solve the model.

The toolkit default options are based on Dynare and on the standard optimal policy problem in DSGE models: users should give the name of one objective variable and one set of policy parameters per player. The algorithm solves the model at second order and uses the second-order steady state of the objective variables as payoff.

The user, however, is free to specify any custom function as objective. This allows: i) to use a first order solution and a LQ approximation to compute the objective functions; ii) players to target *any* objective, such as volatilities or impulse response functions; iii) to use *any* model as input (e.g., non-DSGE models or DSGE models solved without using Dynare) as long as they entail one objective function value per player. When only one player is selected, the toolkit solves the standard optimal policy problem.

Also note that Dynare should be version 4.6 or later for `DSGE Nash` to work.

To install `DSGE Nash` simply add the folder `dsge_nash_file` to the Matlab path. The package is available -and continiously updated- on the authors' websites: Massimo Ferrari Minesso, Maria Sole Pagliari.

`DSGE Nash` is available as free software, under the GNU General Public License version 3. The toolbox comes as it is. We assume no responsibility whatsoever for its use by third parties and make no guarantees, implicit or explicit, about its quality, reliability, or any other characteristic. We would appreciate acknowledgment by citation of the working paper presenting `DSGE Nash` whenever the software is used. We extensively tested the toolbox prior to its public release. Users may inform the authors via email of potential problems with the routines when they emerge.

C.2 Preliminaries

There are two preliminary steps to take before using `DSGE Nash`:

1. If a Dynare mod file is used, users should end it with the `stoch_simul()` command, where the option `order` should be included, so to ensure that the correct order of approximation is set in `DSGE Nash`.

2. If Dynare is not used at all or the computation of the objective function requires other calculations, users should include them in the `user_defined_function.m`. In this file any calculations can be made, provided that the file assigns a value to the players' objective function.

3. The policy parameters need to appear in the mod file or in the user defined objective calculations. Similarly, the objective functions should be assigned a value.

Hint: models used in `DSGE Nash` could be "optimized" to increase computation speed. One way is to substitute redundant equations. For example, when writing the recursive welfare function, $W_t = U_t + \beta E_t(W_{t+1})$, user should directly replace U_t and remove it from the endogenous variables.

Hint: several Dynare options should also be used. Pruning should be selected at higher solution orders. Output should not be printed on screen (`noprint`) and graphs not created (`nograph`). Generally speaking, autoregressive coefficients (`AR=0`), impulse responses (`irf=0`) and simulations (`periods=0`) are not needed, unless explicitly required in the user-defined function. When targeting second-order steady state, the final line of the mod file should look like:

`stoch_simul(order=2,pruning,IRF=0,AR=0,nograph,noprint);`.

`DSGE Nash` is executed by running `dsge_nash(useroptions)`, where `useroptions` is a structure that contains all the settings needed to run the toolkit. If `useroptions` is empty, the toolkit will ask users to input the settings through the terminal.

C.3 Example files

There are 7 example files for `DSGE Nash`:

1. `PrisonersDilemma.m` solves the classic Prisoners' Dilemma game with `DSGE Nash`. It uses the `userdefined` options to set up the payoff matrix of the Prisoners' Dilemma game.

2. `EXAMPLE1.m` solves the game proposed in the companion paper to `DSGE Nash` with parallelization.

3. `EXAMPLE2.m` solves the game under the `userdefined` options, where the objectives of players are the volatilities of some endogenous variables as computed by Dynare.

4. `EXAMPLE3.m` solves the optimal policy problem *without* strategic interactions for the central bank of the model proposed in the companion paper to `DSGE Nash`. The solution is found through a grid search.

5. `EXAMPLE3bis.m` solves the optimal policy problem *without* strategic interactions for the dominant commodity exporter of the model proposed in the companion paper to `DSGE Nash`. The solution is found through a minimization algorithm.

6. `EXAMPLE4.m` runs moment matching with `DSGE Nash`.

7. `EXAMPLE4.m` runs IRFs matching with `DSGE Nash`.

C.4 Settings for policy games based on second order stochastic steady state of DSGE models

In the default setting of `DSGE Nash` the second-order mean of endogenous variables is taken as objective. In a nutshell, the algorithm solves the model for a given combination of policy parameters and extracts the second-order mean of the target variables from the `oo_.mean` structure generated by Dynare. An example of the default settings is available in the example file `EXAMPLE1.m`. Here follows a list of settings:

- `modname`: a string with the mod file name
- `usedynare`: =1 if Dynare is used to solve the model, 0 otherwise
- `nplayers`: number of players
- `parallel`: =1 to parallelize computations, 0 otherwise. The number of workers in the parallel pool should be set in the Matlab preferences
- `userdefined`: =0 if the objective is given by the second-order means of Dynare, =1 if a user-defined function is used to compute the objective
- `ovveridedynare`: =1 to change the default Dynare settings with user-defined settings; 0 otherwise
- user-defined options:
  - `pruning` =0/1 to use pruning;
  - `irf`=0 for no IRFs;
  - `ar`=0 for no AR coefficients;
  - `nograph`=1 for no graphs;
  - `noprint`=1 for not outputing on screen Dynare calculations;
  - `periods`=0 for no simulations.

- `objname`: a cell array containing the names of the objective variables for each player, e.g. example {`‘OBJ1’;‘OBJ2’`}

- `instname`: a cell array with the list of instruments for each player, e.g. `instname{1,1}` `={‘INST1_1’,‘INST2_1’}` for the first player and `instname{2,1}={‘INST1_2’}` for the second player

- `grid`: grid of values for each parameter, e.g. `grid{1,1}(1,1)={[0:0.2:4]}` and `grid{1,1}(1,2)={[0:0` for the first player and `grid{2,1}(1,1)={[0:0.05:0.8]}` for the second player

- `exclusion_condition`: a cell array containing conditions for the parameters whereby the model is not solved and NaN are reported for all players' payoffs. For example, this option can be used to solve the model only under the Taylor principle. Each condition should be stated in a different row of the cell array as a logical condition in text format, e.g `exclusion_condition{1,:}=[‘PHI_HEAD+PHI_COMM+PHI_CORE+PHI_Y<1’];` makes sure that only models with the Taylor principle are considered. A second condition could be added as `exclusion_condition{2,:}=[‘[(PHI_HEAD>0) + (PHI_CORE>0)]==2’]`. Notice that Matlab cannot process multiple logical statements in the same line. For this reason, conditions requiring more than one logical statement (as in the previous example) should be written as sums of the single logical statements. If the condition is a logical "and" (i.e. all statements need to be simultaneously true), this sum should equal the number of conditions. If the condition is a logical "or" (i.e. only one of the conditions should be true) the sum should be equal to 1. The same logic applies to different types of logical statements. For example, if at least one condition should hold the sum should be larger than 1 and so on.

`DSGE Nash` performs a grid search to fill the payoff matrix, i.e. to compute the payoff of each player given a specific combination of strategies. Notice that when a player can choose more than one instrument, a strategy for that player is given by a value for each of his instruments. When using the option `exclusion_condition`, the algorithm automatically excludes any irrelevant convolution of parameters, i.e. not satisfying the restrictions.

Hint: adding some `exclusion_condition` reduces the computation time of the code as irrelevant convolutions of parameters are excluded. These conditions are checked both in the preamble of the code when strategies are listed and at each iteration of the algorithm. Too many conditions in `exclusion_condition`, therefore, might slow down computations because of the excessive evaluation of logical conditions imputed as strings at each iteration of the algorithm.

C.5 Settings for policy games based on other user-defined objective functions

Instead of setting as objective the second-order mean of endogenous variables (the standard choice in optimal policy problems), users can decide to compute themselves the objective variable for some of the players. Examples of target variables include: the volatility of variables, specific impulse responses, the LQ approximation of endogenous variables if available.[25]

These user-defined target variables should be computed in `user_defined_function.m`. As an example, in `EXAMPLE2.m` the objective is a weighted average of inflation and output volatility. Settings are identical to the previous case with the exception of:

- `userdefined`: =1
- `simlist`: a cell array of variables that Dynare needs to simulate the user-defined calculations. By default `DSGE Nash` simulates only the players' target variables. If users choose a user-defined simulation and need to simulate a set of variables from the model, they should list them in this field. For example `simlist = {'NY';'NPI';'NPE'}` . To simulate all variables in the model use `simlist = { }`
- if IRFs need to be computed, then `irf>0`; if data simulations are needed `periods>0`
- if users do not want to use Dynare at all, they should put all the computations required to get the value of the objective variables in the file `user_defined_function.m` and select `usedynare=0`

Hint: if second order solutions are not needed, users should end the mod file by setting `stoch_simul(order=1)`.

C.6 Settings for simple optimal policy exercise

If only one player is selected, `DSGE Nash` re-casts the problem as an optimal policy problem. The code computes the value of parameters that maximise the value of the objective variables. This is the standard welfare-optimization exercise. `EXAMPLE3.m` provides an example using this option. In addition to the baseline options, users need to provide:

- `usemin`: =1 if users want to use a minimization algorithm to solve the policy problem (in this case the min and max values of the parameters' grid are use as bounds for the optimizer); =0 if users want to use a grid search

[25] This has the advantage of solving the model at first order only, thus making computations significantly faster.

- `minoptions`: options for the optimizer; users can set any options available in `Matlab`. Default is `optimset('Algorithm','interior-point','AlwaysHonorConstraints','bounds', 'TolFun', 1e-5,'TolX', 1e-5,'MaxFunEvals',100000000,'MaxIter',100000000, 'PlotFcns',@optimplotfval)`

Hint: also in this case users can compute themselves the target variable by using user-defined computations either based on the Dynare output (`userdefined=1`, `usedynare=1`) or not (`userdefined=1`, `usedynare=0`).
Hint: in case the optimizer is used, users can specify ranges for each policy variable using the `grid` option. The maximum and minimum value for each grid will be used as boundaries.

C.7 Settings for IRFs or moment matching

`DSGE Nash` allows to perform impulse response functions or moment matching based on the impulse responses or moments set by Dynare. Users can specify the options for the IRFs or moment computations using `DSGE Nash` options or directly into the Dynare file. Moreover, users need to store in the working directory a mat file containing the moments or IRFs to match. Both matching algorithms make use of a Newton-based minimization of the distance between the empirical and model-generated data. Options for the algorithms can be specified through `minoptions`. The initial values are taken from the mod file calibration. Examples of moment and IRFs matching are provided in `EXAMPLE4.m` and `EXAMPLE5.m` respectively.
IRFs matching
Users need to save empirical IRFs to be matched with the model-generated IRFs in a Matlab file located in the current working directory. Empirical IRFs should be named exactly as the IRFs generated by the model, i.e. `variablename_shockname`. Additional options needed for IRFs matching are the following:

- `irfmatch`: =1 this option starts the IRF matching algorithm
- `matchname`: a string array containing the name of the file with the empirical IRFs
- `irf_list`: a cell array containing the list of the IRFs to be matched, as produced and stored by Dynare in the `oo_.irfs` structure. Example: `irf_list = {'Y_ea';'pi_ea';'C_ea'}`

Hint: users can use `DSGE Nash` options suche as `irf`, `nomoments`, `periods` to override Dynare options, but it is simpler to just set them in the users' mod file.
Hint: users should restrict the length of the IRFs only to the maximum horizon considered in the empirical exercise. While this is done automatically by `DSGE Nash`, it makes computations

faster when higher order expansions with pruning are used.

Moments matching

Users need to save empirical moments to be matched with model-generated data in a Matlab file located in the current working directory. `DSGE Nash` allows to match both volatilities and averages. Users should list the names of the variables whose volatility should be matched via the option `matchname_std`. The empirical variance covariance matrix of all the variables listed in `matchname_std` should be stored in the Matlab file and called VCOV. The option `momentweight` allows to target only a subsample of cross-correlations of the listed variables. Similarly, users should list the names of the variables whose average is targeted with the option `matchname_mean`. The empirical means of those variables should be stored in the Matlab file in the vector AVG. Additional options needed for moments matching are the following:

- `momentmatch`: =1 this options jump starts the moment matching algorithm
- `matchname`: a string array containing the name of the file with the empirical IRFs
- `matchname_std`: a cell array containing the list of the variables whose variance or correlation is to be targeted
- `momentweight`: a squared matrix to select which moments of the variables listed in `matchname_std` are to be considered. It should have the same dimensions as the empirical variance covariance matrix VCOV. A 0 indicates that the moment is not to be considered while a 1 indicates that it should be targeted. It can also be used to assign weights to different moments by using numbers between 0 and 1
- `matchname_mean`: a cell array containing the list of the variables whose mean is targeted
- `momentweight_mean`: vector used to assign weights to different means. If all variables should have the same weight fill in with ones.

Hint: users can target both volatilities and averages at the same time. In case of moments matching it is best practice to suppress the computations of IRFs and simulations.

Hint: options `nomoments` should be set to 0, otherwise no moments of endogenous variables will be computed.

C.8 Output

In the case of a policy game, all Nash equilibria are printed on screen, i.e. the set of equilibrium strategies and players' payoffs. In case a simple optimal policy exercise is run, the output

will consist of the optimal parameter values. In case of IRFs or moment matching, matched parameters and matching statistics are stored in the output file.
Results are stored in the current working directory. `OutputGridSearch.mat` contains the grid search output (i.e. the payoff matrix). Each player's optimal response function (i.e. the best response given a specific combination of other players' strategies, which are called O_RES1, O_RES2...) is stored in `OptimalResponseFunctions.mat`. `N_OUT.mat` is a structure containing the `DSGE Nash` output.

C.9 Options

Here follows a list of all options that can be specified in the structure `dsge_nash(useroptions)`.

- `ar`: length of AR cofficients [default: =0]
- `exclusion_condition`: a cell array containing conditions for the parameters whereby the model is not solved and NaN are reported for all players' payoffs. Each condition should be a different logical statement in the form of a text string. Check the manual for instructions on how to cast these statements.
- `instname`: cell array with names of instruments for each player
- `irf`: IRFs length [default: =0]
- `irfmatch`: =1 to start the IRFs matching algorithm
- `irf_list`: a cell array containing a list of the IRFs to be matched. In this list IRFs should be named using the Dynare convention `variablename_shockname`; empirical IRFs in `matchname` should have the same name
- `keepout`: =1 to keep all simulation results in the final structure, =0 otherwise
- `matchname`: string containing the name of the Matlab file where empirical IRFs or moments are stored
- `matchname_mean`: a cell array containing the list of the variables whose mean is targeted by the matching algorithm. Such list should use the same names as in Dynare
- `matchname_std`: a cell array containing the list of the variables whose variance or correlation is targeted by the matching algorithm. Such list should use the same names as in Dynare

- `minoptions`: optimizer options [default: `optimset('Algorithm','interior-point', 'AlwaysHonorConstraints','bounds','TolFun', 1e-5,'TolX', 1e-5,'MaxFunEvals', 100000000,'MaxIter',100000000,'PlotFcns',@optimplotfval)`]

- `modname`: string indicating the Dynare mod file name

- `momentmatch`: =1 to start the moment marching algorithm

- `momentweight`: a squared matrix to select which moments of the variables listed in `matchname_std` are to be considered. It should have the same dimension of the empirical variance covariance matrix VCOV. A 0 indicates that the moment is not to be considered while a 1 indicates that it should be targeted. It can also be used to assign weights to different moments by using numbers between 0 and 1

- `momentweight_mean`: vector used to assign weights to different means in the moment matching algorithm. If all variables should have the same weight fill in with ones

- `nograph`: no graph option [default: =1]

- `nomoments`: suppresses computantion of endogenous variables moments by Dynare [default: =0]

- `noprint`: no graph print [default: =1]

- `nplayers`: number of players

- `objname`: cell array containing the names of the objective variables of each player

- `ovveridedynare`: =1 to override Dynare options, =0 (default) otherwise

- `parallel`: =1 to parallelize computations, =0 (default) otherwise

- `pruning`: =1 (default) for pruning, =0 otherwise

- `seed`: random number seed [default: =1999]

- `simlist`: a cell array of variables to be Dynare needs to simulate for the user-defined calculations

- `usedynare`: =1 (default) to use Dynare to solve the model, =0 otherwise. In this case the model solution and the assignment of objective variables needs to be done in `user_defined_function.m`

- `usemin`: =1 (default) to use a minimization algorithm when single-player optimal policy, =0 for grid search
- `userdefined`: =1 if the objective variables are computed manually based on the Dynare ouput in the file `user_defined_function.m`, =0 (default) otherwise