WORKING paper

BANQUE DE FRANCE
EUROSYSTÈME

# Fiscal and Monetary Regimes: A Strategic Approach

**Jean Barthélemy[1] & Guillaume Plantin[2]**

December 2019, WP #742

ABSTRACT

This paper develops a full-fledged strategic analysis of Wallace's "game of chicken". A public sector facing legacy nominal liabilities is comprised of fiscal and monetary authorities that respectively set the primary surplus and the price level in a non-cooperative fashion. We find that the post 2008 feature of indefinitely postponed fiscal consolidation and rapid expansion of the Federal Reserve's balance sheet is consistent with a strategic setting in which neither authority can commit to a policy beyond its current mandate, and the fiscal authority moves before the monetary one at each date.

Keywords: Fiscal Policy, Monetary Policy, Policy Interactions, Game of Chicken

JEL classification: E50 ; E42 ; E63 ; C72

[1] Banque de France, 31 rue Croix des Petits Champs, 75001 Paris, France. Monetary policy and financial research dept. Email: jean.barthelemy@banque-france.fr

[2] Sciences Po and CEPR, 28 rue des Saints-Peres, 75007 Paris, France. Email: guillaume.plantin@sciencespo.fr%20

We thank Marco Bassetto, Roberto Billi, Régis Breton, John Cochrane, Hugues Dastarac, Gaetano Gaballo, Jordi Gali, Eric Mengus, Athanasios Orphanides, Ricardo Reis, Rafael Repullo, François Velde, and participants in various seminars and conferences for helpful comments.

# NON-TECHNICAL SUMMARY

Policy responses to the 2008 crisis have resulted in a massive increase in the liabilities of many fiscal and monetary authorities. In the face of stretched public finances, the question of the interdependence of fiscal and monetary policies has come back to the foreground of policy discussions.

Monetary economists often use a game-theoretic terminology to describe this interdependence. This can be traced back to Wallace's view of a "game of chicken" played among branches of government that control separate elements of the budget constraint. Yet despite these informal references to game theory, this interaction between fiscal and monetary policies has not to our knowledge lent itself to a thorough strategic analysis. The goal of this paper is to develop a general and agnostic strategic analysis of this interaction.

We model Wallace's "game of chicken" as...a game between a fiscal and a monetary authority that jointly face legacy public liabilities. We posit that each authority controls a policy instrument. The fiscal authority sets the real fiscal surplus and the monetary one sets the price level at each date. Both authorities can also trade nominal intertemporal claims---government debt and remunerated reserves, respectively---with the private sector. Each authority incurs costs when its respective policy instrument strays away from a target. Both also incur costs from outright sovereign default.

Our goal is to characterize the equilibria resulting from each set of assumptions on payoffs and strategic interactions, and from there to back out the set of assumptions that delivers the most plausible empirical implications. This way we seek insights into the question that Sargent and Wallace (1981) raise in conclusion of their unpleasant arithmetic: "The question is, Which authority moves first, the monetary authority or the fiscal authority? In other words, Who imposes discipline on whom?"

We start with a simple static version of the game that generates two main insights. First, this game is an actual "game of chicken" only when default costs are convex: accommodation by one authority makes playing tough more appealing to the other. The fixed default cost that is commonly assumed in the literature on sovereign default introduces by contrast a coordination motive among authorities: Accommodation by one of them may make accommodation by the other relatively more attractive (see the figure below). As a result, pure-strategy equilibria with and without default coexist in the presence of fixed. Second, we find that simultaneous games have unappealing properties that limit their practical relevance. We conclude that sequential games are more practically relevant theoretical tools. But, selecting the most relevant game among these requires an answer to Sargent and Wallace's question about who moves first.

In order to answer this question, we then study a dynamic game. Our results are twofold. First, the fiscal authority can force the monetary one to depart from its target and inflate debt away even when the monetary authority moves first at each period. To do so, it needs to roll over government debt until the outstanding amount is sufficiently large that the monetary authority cannot impose full fiscal accommodation, as the fiscal authority would prefer to default in this case. Second, in order to commit to inflation levels that are ex-ante desirable but that would be ex-post excessive, the monetary authority expands its balance sheet by creating sufficiently large amounts of reserves in order to purchase publicly held government debt. This is a credible commitment to future inflation as low future price levels would be inconsistent with the central bank's solvency in (the out-of-equilibrium) case of sovereign default.

We overall conclude that the version of the model in which the fiscal authority moves first is best suited to describe US public finances since 2008. First, this version is the only one in

which the central bank significantly and immediately expands its balance sheet whereas the government postpones fiscal consolidation to the longer run. Second, this version is the only one in which inflation is also postponed to the long-run. Overall, our strategic analysis thus suggests that the answer to Sargent and Wallace's question is: the fiscal authority moves first.

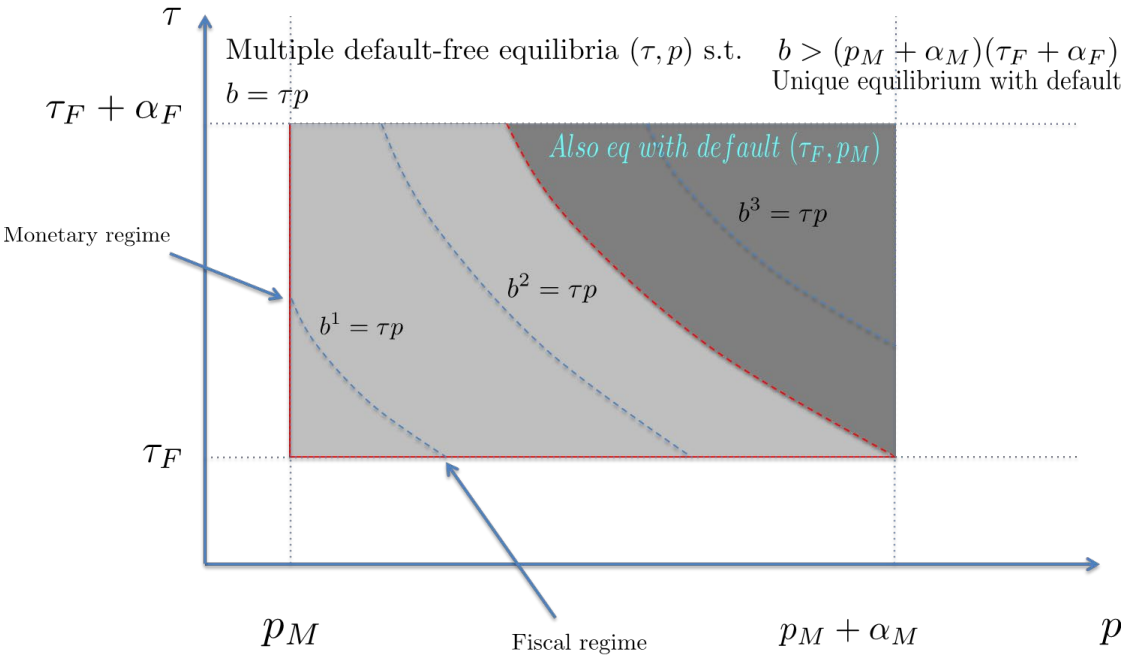


Figure: The Wallace's game of chicken in a static setup

Note: The dashed line describe the set of equilibria for three levels of nominal legacy debt, b, when fiscal and monetary authorities move simultaneously. When b is small multiple of default*free equilibria coexist (light grey), when b is larger there also exists an equilibrium with default (dark grey). For very large b, there exists a unique equilibrium with default. Monetary (fiscal) authority sets the price level p (the real fiscal surplus $\tau$) and targets a level $p_M$ ($\tau_F$ resp.). Payoffs are linear in the instrument and a fixed cost of default $\alpha_M$ ($\alpha_F$ resp.). The no-default condition corresponds to $b = \tau p$.

# Régimes budgétaire et monétaire : Une Approche stratégique

## RÉSUMÉ

Ce papier développe une analyse stratégique du jeu de « la poule mouillée » de Wallace. Le secteur public doit rembourser une dette nominale héritée du passé et est composé d'une autorité monétaire et d'une autorité budgétaire qui ne coopèrent pas. La première choisit le niveau des prix, la seconde le niveau du surplus primaire. Nous montrons que la situation post 2008 aux États-Unis, caractérisée à la fois par un report indéfini de l'ajustement budgétaire et par l'augmentation rapide du bilan de la Réserve fédérale, est cohérente avec (i) aucune autorité ne s'engage au-delà de son mandat courant ; (ii) l'autorité budgétaire joue avant l'autorité monétaire à chaque date.

Mots-clés : Politiques Budgétaire et Monétaire, Interactions Stratégiques, Jeu de la Poule Mouillée

# 1 Introduction

Policy responses to the 2008 crisis have resulted in a massive increase in the liabilities of many fiscal and monetary authorities. In the face of stretched public finances, the question of the interdependence of fiscal and monetary policies has come back to the foreground of policy discussions. At the origin of this longstanding question is the simple observation that the intertemporal budget constraint of the public sector imposes a restriction on the joint path of (nominal) public liabilities, (real) fiscal surpluses, and price levels. As Sargent (1986) simply puts it: *"Arithmetic makes the strategies of the monetary and fiscal authorities interdependent."*

Monetary economists often use a game-theoretic terminology to describe this interdependence. This can be traced back to Wallace's view of a "game of chicken" played among branches of government that control separate elements of the budget constraint.[1]

Yet despite these informal references to game theory, this interaction between fiscal and monetary policies has not to our knowledge lent itself to a thorough strategic analysis. Broadly speaking, the macroeconomic literature has taken two routes to simplify away this interaction. A first route consists in summarizing the behavior of each authority with a policy rule. Monetary and fiscal rules must then be consistent with equilibrium conditions including

[1]Examples abound. Recent ones include Svensson (2017): *"In spite of this interaction, normally monetary policy and fiscal policy are conducted separately, with each policy taking the conduct and effects of the other policy into account. This corresponds to a so-called Nash equilibrium in game theory, where each player chooses his or her instruments independently to achieve his or her goals, while taking into account the conduct of the policy by the other player."* See also Sims' 2013 presidential address: *"(...)With recent repeated Congressional games of chicken over the debt limit and inability to bargain to a resolution of long-term budget problems, the answer may now be in some doubt."*

the non-explosiveness of inflation and public debt (e.g., Leeper 1991). The second route posits that one authority has a free hand at policy making, and so the other authority has no option but accommodating it in order to ensure that the public sector is solvent. In the monetary regime, the monetary authority has a free hand at setting inflation or price levels. It is fully backed by a fiscal authority which, taking price levels as given, ensures that the present value of real government liabilities matches that of primary surpluses. In the fiscal regime of Sargent and Wallace (1981), conversely, the monetary authority sets seignorage revenues, thereby giving up control of inflation, to compensate any gap between real government liabilities and the present value of future primary surpluses both set by the fiscal authority. The goal of this paper is to develop a more general and agnostic strategic analysis of this interaction.

We model Wallace's "game of chicken" as...a game between a fiscal and a monetary authority that jointly face legacy public liabilities. We posit that each authority controls a policy instrument. The fiscal authority sets the real fiscal surplus and the monetary one sets the price level at each date. Both authorities can also trade nominal intertemporal claims—government debt and remunerated reserves, respectively—with the private sector. Each authority incurs costs when its respective policy instrument strays away from a target. Both also incur costs from outright sovereign default.[2]

We do not offer particular micro-foundations for these costs. Our approach is instead one of "revealed preferences". We model each authority's

[2]Costs from outright default in practice include output losses due to financial-market exclusion, trade sanctions, banking crises and more generally financial instability, as well as private costs—electoral or more generally political costs for the fiscal authority and career concerns for central bankers.

payoff in a very flexible and general fashion. We also consider several strategic interactions between them (simultaneous game, sequential game) as well as various degrees of commitment and information structures. Our goal is to characterize the equilibria resulting from each set of assumptions on payoffs and strategic interactions, and from there to back out the set of assumptions that delivers the most plausible empirical implications. This way we seek insights into the question that Sargent and Wallace (1981) raise in conclusion of their unpleasant arithmetic: *"The question is, Which authority moves first, the monetary authority or the fiscal authority? In other words, Who imposes discipline on whom?"*

Section 2 starts with a simple static version of the game that generates two main insights. It first highlights how the structure of default costs affects the nature of the game between authorities. This game is an actual "game of chicken" only when default costs are convex. In this case accommodation by one authority makes playing tough more appealing to the other. The fixed default cost that is commonly assumed in the literature on sovereign default introduces by contrast a coordination motive among authorities: Accommodation by one of them may make accommodation by the other relatively more attractive. As a result, pure-strategy equilibria with and without default coexist in the presence of fixed costs, whereas they do not when costs are convex. Second, and more technically, we find that simultaneous games have unappealing properties that limit their practical relevance. They either feature many equilibria and so lack predictive power or, in a version with uncertainty, counterfactually predict that the public sector frequently resorts to small defaults. We conclude that sequential games such as those implicit

in the monetary and fiscal regimes described above are more practically relevant theoretical tools. But then, selecting the most relevant game among these requires an answer to Sargent and Wallace's question about who moves first.

In order to tease out more testable implications from various assumptions regarding which authority moves first, Section 3 then studies a dynamic game. We make the natural assumption that the public sector incurs immediate costs of default as soon as the path of public finances is not sustainable and regardless of the timing of actual default. This reflects the view that, in particular for developed economies, costs of default such as the loss of a convenience yield or financial instability materialize as soon as the government debt loses its status as a risk-free and liquid asset. This assumption implies that the relative preference of the public sector for fiscal or/and monetary adjustments over default is stronger when these adjustments are more remote. The resulting time inconsistency has two important implications. First, the fiscal authority can force the monetary one to depart from its target and inflate debt away even when the monetary authority moves first at each period. To do so, it needs to roll over government debt until the outstanding amount is sufficiently large that the monetary authority cannot impose full fiscal accommodation, as the fiscal authority would prefer to default in this case. In other words, the fiscal authority gains endogenous control of the price level through strategic fiscal irresponsibility even if the monetary one moves first at each period. Second, in order to commit to inflation levels that are ex-ante desirable but that would be ex-post excessive, the monetary authority expands its balance sheet by creating sufficiently large amounts

of reserves in order to purchase publicly held government debt. This is a credible commitment to future inflation as low future price levels would be inconsistent with the central bank's solvency in (the out-of-equilibrium) case of sovereign default.

We overall conclude that the version of the model in which the fiscal authority moves first is best suited to describe US public finances since 2008. First, this version is the only one in which the central bank significantly and immediately expands its balance sheet whereas the government postpones fiscal consolidation to the longer run.[3] Second, this version is the only one in which inflation is also postponed to the long-run. Overall, our strategic analysis thus suggests that the answer to Sargent and Wallace's question is: the fiscal authority moves first.

## Related literature

As in Aguiar et al. (2013, 2015), we study how the public sector combines inflation, taxes, and outright default in order to cope with a liability shock, and we model the respective costs of these instruments in a reduced form. In Aguiar et al. (2013), the public sector is comprised of one single agent and the focus is on optimal inflation credibility in the presence of self-justified solvency crises. Aguiar et al. (2015) formalize a monetary union as a public sector comprised of a monetary authority and atomistic fiscal authorities. We complement these papers with the study of the case in which both components of the public sector are comprised of strategic agents. The time

[3]Hall (2013) documents a shift away from stabilizing the debt to GDP ratio with fiscal policy that persists long after the post-2008 recession. Recent projections by the Congressional Budget Office (CBO) tend to confirm this shift.

inconsistency induced by our formulation of default costs is another distinctive feature.

An older literature (Alesina 1987, Alesina and Tabellini 1987, Tabellini 1986, and more recently Dixit and Lambertini 2003) study like us Nash equilibria between multiple branches of government. We contribute to this literature in two ways. First, following Kocherlakota (2012), we include sovereign default as a feasible strategy profile. We view this as a pre-requisite to a full-fledged strategic analysis of Sargent and Wallace's unpleasant arithmetic: One needs to specify payoffs when the public sector defaults in order to derive fiscal or/and monetary accommodation along default-free equilibrium paths. In addition, we show how the fiscal and monetary authorities can respectively use government debt and excess reserves as strategic devices to regulate their future influence over the other authority's policy in the absence of commitment power.

Our point that balance-sheet expansion is a device to commit to future price levels that the monetary authority would find ex-post excessive but that are ex-ante desirable echoes several recent rationales for quantitative easing. In Jeanne and Svensson (2007), the depreciation of foreign reserves resulting from low domestic price levels acts as a threat to the solvency of the monetary authority that is similar to ours. In Battharai et al. (2015), the central bank commits to low future interest rates by owning long-term bonds that would depreciate under higher rates, thereby also endangering its solvency. The mechanism that generates a commitment to high future price levels in our setup differs from these, however, and is more strategic in nature. By making its net wealth highly sensitive to the government's

decision to (strategically) default, the central bank deliberately eliminates its future incentives to let the government default. It will prefer to inflate debt away since such a default would come with inflation anyway.

The restrictions that the central bank's solvency imposes on feasible price-level paths rationalize unconventional measures in our setup. These restrictions are also central in the analyses of new-style central banking developed by Hall and Reis (2015) and Reis (2015, 2017). Our strategic approach delivers the novel insight that the swap of government debt with remunerated reserves is not neutral even when both types of claims are risk free and thus perfect substitutes *in equilibrium*. Such expansions of the central bank's balance sheet increase its exposure to the *out-of-equilibrium* threat of sovereign default. This reduces its future credibility, thereby raising future price levels.

# 2 Wallace's game of chicken: Which game?

## 2.1 Setup

We consider a static game between two players, a fiscal authority $F$ and a monetary authority $M$.

**Actions.** The fiscal authority $F$ sets a real primary surplus $\tau \in \mathbb{R}$ and the monetary authority $M$ a price level $p \in \mathbb{R}_+$. Players move simultaneously.

**Payoffs.** $F$ and $M$ face a maturing nominal liability $b \geq 0$. A strategy

profile $(\tau, p)$ yields payoffs $U_X(\tau, p)$ to player $X \in \{F; M\}$ such that

$$U_F(\tau, p) = -g_F\left(|\tau - \tau_F|\right) - f_F\left(\left(\frac{b}{p} - \tau\right)^+\right), \tag{1}$$

$$U_M(\tau, p) = -g_M\left(|p - p_M|\right) - f_M\left(\left(\frac{b}{p} - \tau\right)^+\right), \tag{2}$$

where $(f_X, g_X)_{X \in \{F;M\}}$ are increasing, strictly convex and differentiable functions over $[0, +\infty)$, $\tau_F > 0$, $p_M > 0$, and $x^+ \equiv \mathbb{1}_{\{x \geq 0\}} x$. For conciseness, we restrict the analysis to the case in which $g'_X$ spans $[0, +\infty)$ for $X \in \{F; M\}$.

In words, each authority incurs additively separable costs from i) setting its policy instrument away from a given target[4], and ii) from the (real) loss resulting from sovereign default (equal to 0 if the liability is honored in full).[5] We could extend our approach to the case of non-separability in principle, presumably at the cost of additional analytical complexity. Note that these payoffs (1) and (2) include the case in which $F$ and $M$ share the same objective

$$-g_F\left(|\tau - \tau_F|\right) - g_M\left(|p - p_M|\right) - f\left(\left(\frac{b}{p} - \tau\right)^+\right) \tag{3}$$

but do not cooperate to maximize it.[6]

These payoffs are such that the fiscal and monetary authorities have no reason to deviate from their respective targets other than avoiding to default

[4]The analysis carries through if $M$ has an inflation target.

[5]Results are qualitatively similar if the nominal loss given default matters.

[6]This is because adding to $U_X$ terms that do not depend on $X$'s instrument does not affect its optimal decision. This is only true in this simultaneous-move version of the game though.

on the maturing public liabilities $b$. This implies that there is no point for either authority to undershoot its target as it increases the loss given default. In game-theoretic language, $\tau_F$ dominates any strategy $\tau \leq \tau_F$ for $F$, and $p_M$ dominates any strategy $p \leq p_M$ for $M$. This implies in particular that $(\tau_F, p_M)$ is the unique Nash equilibrium (and is an equilibrium in dominant strategies) when $b \leq \tau_F p_M$. The remainder of the paper focusses on the alternative situation of interest in which $b > \tau_F p_M$.

In this case, if no player accommodates by overshooting its target, then there is outright default. The situation in which the fiscal authority stays on its primary-surplus target whereas the monetary authority fully accommodates to avoid default, i.e., inflates debt away, corresponds to what is commonly described as the fiscal regime in the monetary literature, whereas the symmetric situation corresponds to the monetary regime. In our strategic formulation, each regime corresponds to a particular strategy profile:

**Definition.** ***(Fiscal and monetary regimes)*** *The fiscal regime is the strategy profile* $(\tau_F, b/\tau_F)$ *and the monetary regime is the strategy profile* $(b/p_M, p_M)$.

We now solve for the Nash equilibria of this game, with a particular interest in identifying the circumstances under which either the fiscal regime or the monetary regime are Nash equilibria.

## 2.2 Equilibria

We first focus on equilibria in pure strategies. The functions $U_F(., p)$ and $U_M(\tau, .)$ are strictly concave and bounded above and so admit each a unique

maximizer which is the best response to the other authority's policy. Strict concavity of $U_F$ and $U_M$ also implies that these best responses are decreasing in the other player's action. In other words, $M$ and $F$ play a "game of chicken", whereby accommodation is a strategic substitute. More accommodation by one authority makes playing tough more appealing to the other.

Let us denote $\tau_M$ the surplus beyond which $F$ prefers to default given no monetary accommodation, and $p_F$ the price level beyond which $M$ prefers to default given no fiscal accommodation. Formally,[7]

$$\tau_M = \sup_{\tau \geq \tau_F} \{g'_F(\tau - \tau_F) \leq f'_F(0)\}, \tag{4}$$

$$p_F = \sup_{p \geq p_M} \{p g'_M(p - p_M) \leq \tau_M f'_M(0)\}. \tag{5}$$

The following proposition characterizes the pure-strategy equilibria of the game:

**Proposition 1.** ***(Pure-strategy equilibria)*** *There exists* $\tau_F p_M \leq b_- \leq b_+ \leq \tau_M p_F$ *such that*

- *If* $b \in (\tau_F p_M, b_-]$ *then there exists a continuum of equilibria given by the strategy profiles* $(\tau, b/\tau)$ *where* $\tau \in [\tau_F, b/p_M]$*. Thus there is no default in equilibrium and both the monetary and fiscal regimes correspond to equilibria.*

- *If* $b \in (b_-, b_+]$*, there also exists a continuum of equilibria without default. Only one of the two regimes, either fiscal or monetary depending on parameter values, is an equilibrium outcome.*

[7]Note that $f'_F(0)$ and $f'_M(0)$ must be strictly positive in order for $\tau_M$ and $p_F$ to be strictly above $\tau_F$ and $p_M$ respectively.

- *If $b \in (b_+, \tau_M p_F]$, there also exists a continuum of equilibria without default that include neither the fiscal nor the monetary regime.*

- *If $b > \tau_M p_F$ there exists a unique equilibrium with default such that $p, \tau$, and the loss given default all increase with respect to $b$.*

**Proof.** See the appendix. ■

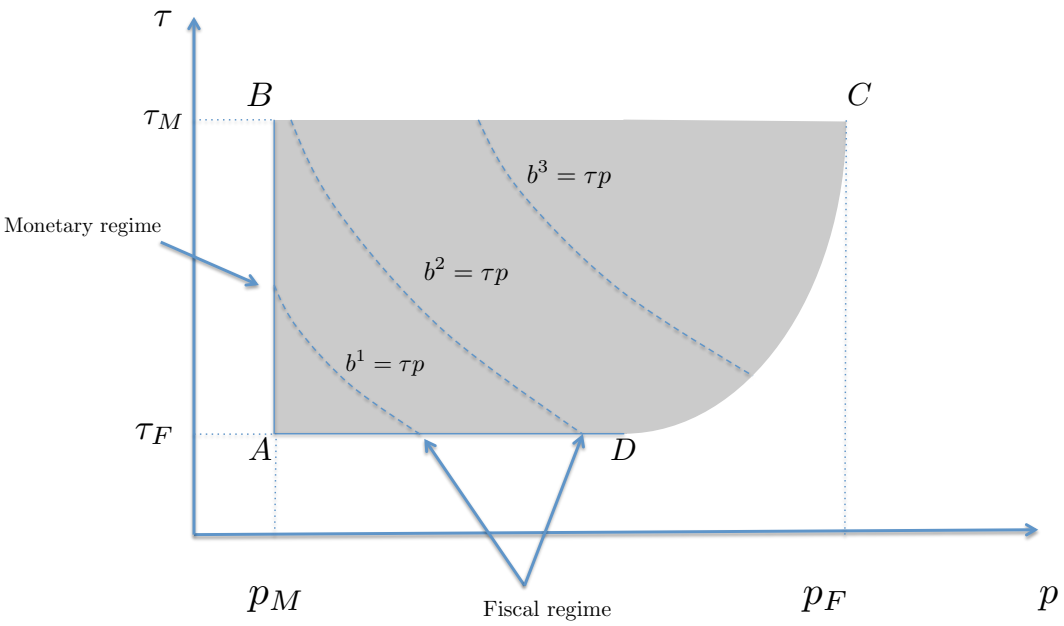


Figure 1. The dashed segments of hyperbolae represent the pure-strategy equilibria associated with three values $b^1, b^2, b^3$ such that $\tau_F p_M < b^1 < b_- < b^2 < b_+ < b^3 < \tau_M p_F$.

The equilibria are best described graphically in the plane $(p, \tau)$. Refer to Figure 1. If $b$ is sufficiently large that the graph of the hyperbola $b = \tau p$ is to the northeast of $C = (p_F, \tau_M)$, then there is a unique equilibrium with default. Otherwise, the equilibria are default-free, described by the

segment of the hyperbola $b = \tau p$ that is within the shaded area ABCD.[8] The intersection of this segment (if any) with AB is the monetary regime whereas that with AD (if any) corresponds to the fiscal regime. For sufficiently low levels of debt, both the monetary and fiscal regimes are equilibrium outcomes. This is illustrated by the case $b = b^1$ in Figure 1. As $b$ increases, one of the two regimes ceases to be an equilibrium outcome. This is illustrated by the case $b = b^2$ in Figure 1 in which the monetary regime is not an equilibrium outcome.[9] Both authorities accommodate as $b$ gets close to the value at which they default (as in the case $b = b^3$ in Figure 1).

Regarding the circumstances under which the fiscal and monetary regimes are Nash equilibria, Proposition 1 has two unsurprising but interesting implications. First, none of these regimes is an equilibrium outcome as soon as public debt is sufficiently large other things being equal, as both parties accommodate in this case. This formalizes the common wisdom that fiscal and monetary coordination is more important the higher the level of public debt.

Second, if the monetary or/and the fiscal regime are rationalizable outcomes, then unfortunately they coexist with a continuum of default-free equilibria in which both parties accommodate.

**Mixed-strategy equilibria.** All pure-strategy equilibria are default-free when $b \leq \tau_M p_F$. Yet there also exists for such values of $b$ a large collection of mixed-strategy equilibria that all feature stochastic default.[10]

[8] This area collapses to A and so there is a unique equilibrium with default if $f'_F(0) = f'_M(0) = 0$.

[9] Either regime can survive in general in this area depending on which authority has the highest cost of default relative to deviation from target.

[10] It is easy to see for example that any pair of (default-free) pure-strategy equilibria

The next section reduces the number of equilibria by adding the ingredient that both authorities have imperfect control over their respective instruments.

## 2.3 Uncertainty

A simple way to reduce the number of equilibria and thus obtain some predictive power is to introduce uncertainty in this game. This section does so by leaving the previous model unchanged, up to the additional assumption that $F$ and $M$ imperfectly control their respective policy variables. If $F$ seeks to set the real surplus at $\tau$, an actual surplus $\tau e^{\epsilon_F}$ is realized. Similarly, the realized price level is $pe^{\epsilon_M}$ if $M$ sets a price level $p$. $F$ and $M$ set $\tau$ and $p$ simultaneously. They share the prior belief that the random variable $(\epsilon_F, \epsilon_M)$ admits a p.d.f. $1/\sigma^2\varphi(\epsilon_F/\sigma, \epsilon_M/\sigma)$, where $\sigma > 0$ and $\varphi$ has a bounded support[11] that includes a neighborhood of $(0, 0)$.[12] $F$ and $M$ seek to maximize their respective expectations over the payoffs defined in (1) and (2). We also assume in this Section 2.3 that the functions $(f_X, g_X)_{X\in\{F;M\}}$ are continuously differentiable.

**Proposition 2.** ***(Uncertainty yields equilibrium uniqueness)*** *For every $\sigma > 0$, there exists a unique equilibrium $(\tau(\sigma), p(\sigma))$. The equilibrium probability of default is strictly positive. If $b > \tau_M p_F$, then $\lim_{\sigma\to 0}(\tau(\sigma), p(\sigma))$ is the strategy profile of the unique equilibrium obtained in the model without*

$(\tau, p)$ and $(\tau', p')$ can be associated with an equilibrium with stochastic default in which $F$ and $M$ mix over the strategies associated with each equilibrium.

[11] This is only meant to abstract from convergence issues.

[12] A positive shock on output is likely both to generate a higher surplus than initially budgeted and to create inflationary pressure. A realistic assumption is thus that $\epsilon_M$ and $\epsilon_F$ be positively correlated, but this plays no role in our analysis.

*uncertainty. If* $b \leq \tau_M p_F$, $\lim_{\sigma \to 0}(\tau(\sigma), p(\sigma))$ *is the unique solution to*

$$b = \tau p, \tag{6}$$

$$p g'_M(p - p_M) f'_F(0) = \tau g'_F(\tau - \tau_F) f'_M(0). \tag{7}$$

*The probability of default converges to* $g'_F(\tau - \tau_F)/f'_F(0)$ *and the loss given default tends to 0 as* $\sigma \to 0$ .

**Proof.** See the appendix. ■

On the one hand, this version of the model delivers a unique equilibrium in which both parties accommodate. On the other hand, accommodation always comes with a strictly positive probability of sovereign default. The loss given default however becomes arbitrarily small for $b \leq \tau_M p_F$ in the limiting case in which uncertainty vanishes. This prediction that the public sector uses small sovereign defaults as a routine tool whenever some accommodation is necessary is highly counterfactual. It is at odds with the commonly held view of a pecking order, whereby sovereign default is a last-resort strategy used only after other options have failed, at least in developed economies.

## 2.4 Non-convex cost of default

We aim at modelling payoffs in an agnostic and flexible fashion, and so we started above with a simple convex cost of default. The sovereign-default literature commonly assumes non-convexities, however, in the form of a fixed component to the cost of default. We introduce here such a fixed cost by

assuming payoffs:

$$U_F(\tau, p) = - \mid \tau - \tau_F \mid - \alpha_F \mathbb{1}_{\{b > \tau p\}}, \tag{8}$$

$$U_M(\tau, p) = - \mid p - p_M \mid - \alpha_M \mathbb{1}_{\{b > \tau p\}}, \tag{9}$$

where $\alpha_F, \alpha_M > 0$.[13] The assumption of a linear dependence in $\mid \tau - \tau_F \mid$ and $\mid p - p_M \mid$ is just a normalization given the fixed cost of default. We let

$$\beta_1 = \inf\{\tau_F(p_M + \alpha_M); (\tau_F + \alpha_F)p_M\}, \tag{10}$$

$$\beta_2 = \sup\{\tau_F(p_M + \alpha_M); (\tau_F + \alpha_F)p_M\}, \tag{11}$$

$$\beta_3 = (\tau_F + \alpha_F)(p_M + \alpha_M), \tag{12}$$

and characterize pure-strategy equilibria:

**Proposition 3.** ***(Pure-strategy equilibria with a fixed default cost)***

- *If $b \in (\tau_F p_M, \beta_1]$ there exists a continuum of equilibria given by the strategy profiles $(\tau, b/\tau)$ where $\tau \in [\tau_F, b/p_M]$. Thus there is no default in equilibrium and both the monetary and fiscal regime correspond to equilibria.*

- *If $b \in (\beta_1, \beta_2]$, there also exists a continuum of equilibria without default. The fiscal regime is an equilibrium but not the monetary one if $\beta_1 = (\tau_F + \alpha_F)p_M$ whereas the opposite holds otherwise.*

- *If $b \in (\beta_2, \beta_3]$, there exists a continuum of equilibria without default*

[13] The addition of a variable cost to the fixed one assumed here does not significantly affect the analysis. Assuming pure fixed costs of default implies that both authorities always prefer to set their instrument at the target conditionally on default.

*that does not include the fiscal nor the monetary regime. There also exists an equilibrium in which $F$ and $M$ default and play their targets $(\tau_F, p_M)$.*

- *If $b > \beta_3$ then this latter equilibrium with default is the unique one.*

**Proof.** See the appendix. ■

As soon as $b$ is sufficiently large that neither the fiscal nor the monetary regime are equilibrium outcomes ($b > \beta_2$), then the continuum of default-free equilibria coexist with the equilibrium with default. This is the salient and interesting difference with the case of a convex cost studied in Proposition 1, in which default-free equilibria do not coexist with the cum-default equilibrium in pure strategies. The intuition for this difference can be found in the strategic impact of the assumed fixed cost. With a fixed cost, the game is no longer a "game of chicken." Fiscal and monetary accommodations are no longer pure strategic substitutes as the fixed cost induces strategic complementarity. The cost of accommodation relative to the cost of default for one authority decreases as the other authority accommodates more because a smaller accommodating effort then suffices to avoid the fixed default cost. This creates a coordination motive between $F$ and $M$ that generates the additional multiplicity of equilibria—default-free versus cum-default.

One can show that as in Proposition 2, the introduction of small exogenous shocks to the policy variables reduces the number of equilibria. There remain however multiple equilibria under uncertainty in this case of a fixed cost. Because of the above-mentioned coordination problem, there still are two equilibria with varying probabilities of default in the limiting case of vanishing uncertainty. Also, the setup with uncertainty still has the unpalatable

feature that default occurs even for small values of $b$.

**A negative progress report.** The analysis suggests that the simultaneous games considered thus far are tools of limited applicability to the study of fiscal and monetary coordination. Without uncertainty, the number of equilibria is too large to generate useful predictions. The addition of uncertainty prunes equilibria, but small sovereign defaults arise as a routine tool in the remaining equilibria, contrary to the evidence that sovereign defaults typically are of significant magnitude. It is also at odds with the commonly held view that default is a "nuclear option" that is not seriously considered below very large debt levels. This motivates the restriction of the rest of the paper to games of sequential moves.

## 2.5 Sequential moves

This section overcomes the issues facing simultaneous games by modelling the interaction between $F$ and $M$ as games of sequential moves in which one authority is the Stackelberg leader. In such sequential games, the convexity of default costs does not have the same important implications as in the case of a simultaneous game, and so we adopt for notational simplicity the fixed-cost setup of Section 2.4. More precisely we assign payoffs (8) and (9) to $F$ and $M$ and consider two games—fiscal lead and monetary lead. Under fiscal lead, $F$ sets the surplus $\tau$ and then $M$ sets the price level $p$ observing this, whereas $M$ is the first mover under monetary lead.[14]

[14]One type of strategic interaction that this paper does not consider is bargaining, as there is no such thing as formal negotiations on fiscal and monetary policy between governments and independent central banks in practice. Each authority may merely issue views on the policy adopted by the other. In any case, it is easy to see that the sequential

**Proposition 4.** ***(Fiscal versus monetary lead)***

- *Under fiscal lead, there is a unique equilibrium:*

$$\begin{cases} (\tau_F, b/\tau_F) & \text{if } b \in (\tau_F p_M, \tau_F(p_M + \alpha_M)], \\ (b/(p_M + \alpha_M), p_M + \alpha_M) & \text{if } b \in (\tau_F(p_M + \alpha_M)), (\tau_F + \alpha_F)(p_M + \alpha_M)), \\ (\tau_F, p_M) & \text{if } b > (\tau_F + \alpha_F)(p_M + \alpha_M). \end{cases}$$

- *Under monetary lead, there is a unique equilibrium:*

$$\begin{cases} (b/p_M, p_M) & \text{if } b \in (\tau_F p_M, (\tau_F + \alpha_F)p_M], \\ (\tau_F + \alpha_F, b/(\tau_F + \alpha_F)) & \text{if } b \in ((\tau_F + \alpha_F)p_M, (\tau_F + \alpha_F)(p_M + \alpha_M)), \\ (\tau_F, p_M) & \text{if } b > (\tau_F + \alpha_F)(p_M + \alpha_M). \end{cases}$$

**Proof.** The leader just picks the simultaneous-game equilibrium that warrants him the highest payoff. Under fiscal lead, as $b$ increases, the equilibrium is first the fiscal regime, $F$ then partially accommodates once $M$ accommodates so much that it is indifferent with default, and finally the public sector defaults. Monetary lead is the mirror image. ■

games that we consider deliver the same outcomes as ultimatum games—bargaining games in which one authority has the entire bargaining power.

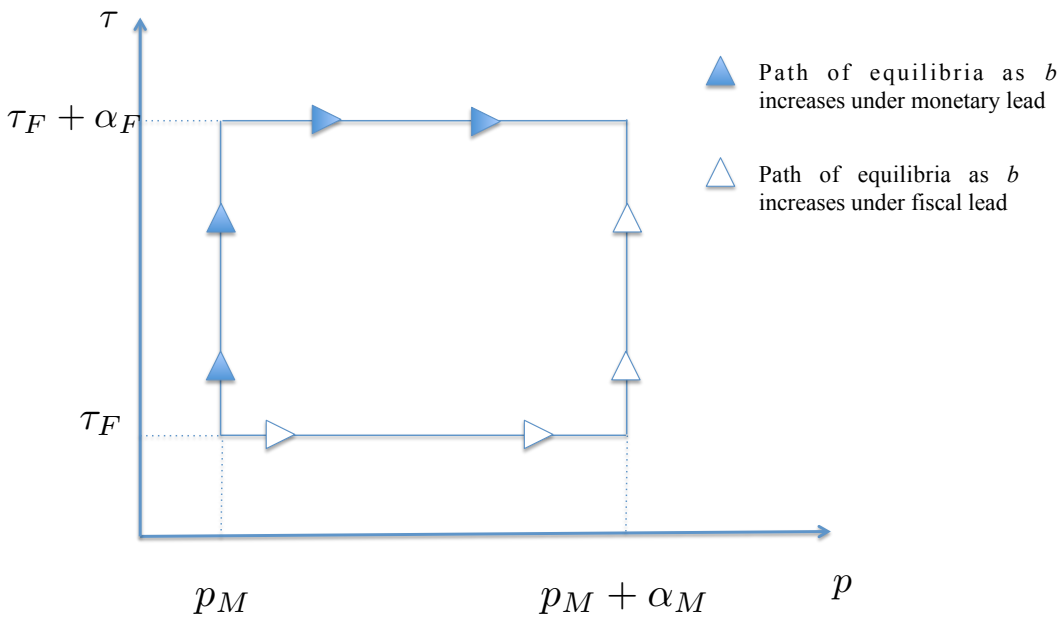


Figure 2. Equilibria under monetary and fiscal lead.

Figure 2 illustrates how the equilibrium evolves with $b$ under monetary and fiscal lead.

**Who moves first?** Sequential games eliminate the issues of equilibrium indeterminacy or implausibility facing simultaneous games: There is a unique equilibrium which is default-free below a threshold for $b$. But this comes of course at the cost of making a non-obvious assumption regarding the first mover. In other words, if one could derive the objectives (8) and (9) of $F$ and $M$—or a more sophisticated version of them—out of a structural model of the economy, then one would still be left with little empirical guidance as to the exact structure of their interaction. One way to obtain some guidance is to tease out distinctive empirical implications from various assumptions about the order of moves. We now show that a dynamic version of this game

with a richer debt structure delivers such implications.

# 3 Dynamic game of chicken: Public liabilities as commitment devices

This section studies dynamic versions of the simple games with sequential moves and fixed default costs studied in Section 2.5.

## 3.1 Setup

Time is discrete and is indexed by $t \in \mathbb{N}$. At each date $t$, the fiscal authority $F$ sets a real primary surplus $\tau_t \in \mathbb{R}$ and the monetary authority $M$ a price level $p_t > 0$.[15]

**Endogenous public liabilities.** In addition to setting their respective policy instruments at each date, $F$ and $M$ can also promise nominal future payments to a private sector comprised of competitive risk-neutral investors who discount future consumption with a factor $\beta \in (0, 1)$. We suppose that optimization by the private sector implies a transversality condition,[16] and so the value of public liabilities must be equal to investors' expected present value of future surpluses.[17] The fiscal authority $F$ is free to trade any

[15]There are (at least) two simple cashless environments in which the monetary authority can set the price level at each date this way. In the cashless limit of the cash-in-advance model set up in Cochrane (2005), it does so by supplying the appropriate amount of intra-date money. In Hall and Reis (2017), it does so by promising an appropriate real repayment on one-period reserves.

[16]This is for example the case if the private sector is comprised of identical infinitely-lived agents with a bounded exogenous endowment.

[17]Hall and Reis (2015) and Reis (2015, 2017) discuss the implications from a central bank's insolvency under the same assumption. Barthélemy et al. (2018) study the case in

stream of fixed promised repayments with the private sector. All repayments promised by $F$ at a given date are pari-passu.[18] The monetary authority $M$ can purchase outstanding government securities held by the private sector by issuing nominal claims with arbitrary repayment schedules. We deem these liabilities issued by the central bank "remunerated reserves". $F$ and $M$ make take-it-or-leave-it offers of prices and quantities to the private sector.[19]

Central bank reserves are the fundamental unit of value in every modern economy. A haircut on excess reserves is thus basically a currency reform. Such radical reforms clearly exceed the scope of central banks' mandates. We capture this in our setting by assuming that $M$ finds any option preferable to a default on remunerated reserves.[20]

**Assumption 1.** ***(Default on reserves)*** *The monetary authority $M$ incurs an arbitrarily large disutility from defaulting on remunerated reserves.*

We suppose that $M$ receives an exogenous real income stream with date-1 present value $\epsilon > 0$.[21] For expositional simplicity, we first study the limiting case in which $\epsilon \downarrow 0$, and so this income has a negligible effect on the primary surplus if paid out as a dividend to the government. We then address the case of a finite $\epsilon$ in Section 3.5.

**Payoffs.** A course of actions by $F$ consists in real surpluses $(\tau_t)_{t\geq 0}$ and a set of promises of date-$t'$ nominal repayments issued at date $t$, $(b_{t,t'})_{t'>t\geq -1}$. The

which public liabilities can be bubbly.

[18] This plays no other role than simplifying the exposition.

[19] This rules out equilibrium multiplicity due to multiple self-justified prices in a Walrasian bond market, as in Aguiar et al. (2013). The unique equilibrium that we obtain would still be an equilibrium in a Walrasian version of the model.

[20] Section 3.5.4 shows that whether so does $F$ or not does not affect our main results.

[21] Central banks' income is in practice mainly comprised of seignorage—our environment is cashless, however—and of return from assets possibly purchased at distressed prices.

actions of $M$ are symmetrically comprised of price levels $(p_t)_{t\geq 0}$ and a set of date-$t'$ nominal repayments promised at date $t$, $(r_{t,t'})_{t'>t\geq -1}$. Promises issued at date $t=-1$ represent exogenous liabilities inherited from an unmodelled past (the counterpart of $b$ in the static model of Section 2).

Let $\Delta_t$ denote an indicator function equal to 1 if and only if a strategy profile implies default on government debt held by the private sector after date $t$.[22] The respective date-$t$ payoffs of $F$ and $M$ are then

$$V_t^F = -\sum_{s\geq 0} \beta^s \mid \tau_{t+s} \mid -\beta(1+\beta)\alpha_F \Delta_t, \tag{13}$$

$$V_t^M = -\sum_{s\geq 0} \beta^s \mid p_{t+s} - p_M \mid -\beta(1+\beta)\alpha_M \Delta_t, \tag{14}$$

where $\alpha_F, \alpha_M, p_M > 0$.

**Default cost.** These payoffs encode the natural and important assumption that the public sector incurs an immediate cost of default at date $t$ as soon as the strategy profile from date-$t$ on implies default on government debt held by the private sector, no matter how remote the date of this default. This reflects that in advanced economies, government debt plays an important role as a liquid and safe asset (see, e.g., Krishnamurthy and Vissing-Jorgensen 2012 for the US), and so a perfectly foreseeable non-sustainable path has immediate costs (such as financial instability). The factor $\beta(1+\beta)$ applied to the cost of default of $F$ and $M$ is just a convenient normalization.[23]

[22] Formally, $\Delta_t = 1$ if and only if there exists a date $t' \geq t$ at which there is government debt held by the public and the public sector fails to fully honor it ($\sum_{i<t'} b_{i,t'} > \tau_{t'} p_{t'} + \sum_{j>t'} (q_{t',j} p_{t'}/p_j) b_{t',j}$, where $q_{t',j}$ is the (real) date-$t'$ price asked for a date-$j$ unit claim).

[23] The reader may wonder why the private sector does not also incur costs from sovereign defaults beyond lost payoffs, such as the loss of a convenience yield on government debt. This is for conciseness as such costs would not affect the equilibria. Given the assumed

Two further remarks on these payoffs (13) and (14) are in order. First, the target of the fiscal authority is normalized to zero in (13), and so it must accommodate to avoid default as soon as the public sector inherits some strictly positive liabilities at date 0 (as soon as $b_{-1,t} \geq 0$ with a strict inequality for some $t \geq 0$).[24] Second, the public sector discounts future adjustments with the same factor $\beta$ as that of the private sector.

**Initial public finances.** We assume that at the outset, the public sector faces two identical exogenous government debt repayments $b$ at dates 1 and 2. Formally, $b_{-1,1} = b_{-1,2} = b > 0$ and, for all $t \notin \{1; 2\}$, $b_{-1,t} = 0$. There are no initial outstanding reserves ($r_{-1,t} = 0$ for all $t \geq 0$). Any equilibrium path without default must therefore be such that the public sector satisfies a date-0 intertemporal budget constraint:

$$\frac{\beta b}{p_1} + \frac{\beta^2 b}{p_2} \leq \sum_{k \geq 0} \beta^k \tau_k. \tag{15}$$

This debt structure generates all the important insights while remaining analytically tractable.[25]

**Strategic interactions.** We will compare the equilibria from games corresponding to different assumptions regarding order of moves—who leads at each period—and commitment power.

fixed costs of default, the public sector repudiates all outstanding debt at the outset, does not trade new debt, and sticks to its policy targets forever in equilibria with default.

[24] Assuming a strictly positive surplus target clutters the exposition without adding insights.

[25] Note that the static model studied in Section 2 corresponds to $b_{-1,0} = b$ and $b_{t,t'} = r_{t,t'} = 0$ otherwise. So this dynamic model departs from it in two ways: The initial public liability is comprised of two instalments, and nothing is due at date 0.

*Fiscal versus monetary lead.* Under fiscal lead, at each date $t$, $F$ sets $\tau_t$ and possibly issues bonds. Observing this, $M$ sets $p_t$ and possibly issues reserves. $F$ and $M$ swap these positions under monetary lead.

*Commitment power.* Under full commitment, $F$ and $M$ can commit to any plan of future actions. Under limited commitment, they optimize at each date, and so the Nash equilibria resulting from their interactions are subgame perfect. Whereas the full-commitment case is a useful benchmark, we clearly view that of limited commitment as the empirically relevant one. A period in our model is a time interval over which the effects of a given fiscal or/and monetary policy fully materialize. It is thus best interpreted as a period beyond which it is difficult for a given government or central bank leadership with a fixed term to credibly commit. We study in turn monetary and fiscal leads in this context.

The following Section 3.2 first characterises the equilibria under monetary lead (with and without commitment). Section 3.3 then tackles the case in which $F$ leads. Section 3.5 discusses the results. For brevity, the paper describes the important characteristics of the equilibria in a heuristic fashion. An online appendix offers a complete description and rigorous derivation of the equilibria of the games that we consider.

Throughout the analysis, we suppose that $\beta$ is above a given threshold. The online appendix defines this threshold explicitly, and shows that the broad qualitative insights carry through for lower values of $\beta$.

## 3.2 Monetary lead

The following proposition first characterizes the equilibrium under the benchmark of full commitment as the common size $b$ of the legacy liabilities at dates 1 and 2 varies.

**Proposition 5.** *($M$ **leads, full commitment** )*

1. *If $b \leq \alpha_F p_M$ then $M$ announces $p_t = p_M$ for all $t$, and $F$ raises a stream of surpluses with date-0 present value $\beta(1+\beta)b/p_M$.*

2. *If $\alpha_F p_M < b \leq \alpha_F(p_M + \alpha_M)$, then $M$ announces $p_1 = p_2 = b/\alpha_F$, $p_t = p_M$ for $t \notin \{1;2\}$. The fiscal authority $F$ raises surpluses with date-0 present value $\beta(1+\beta)\alpha_F$. It is thus indifferent with default.*

3. *Otherwise $F$ and $M$ default and play their target at each date.*

**Proof.** Propositions E.1 and E.5 in the online appendix. ■

Not surprisingly, the equilibrium is very similar to that in the static game described in Proposition 4. In case *1.*, $M$ forces $F$ to bear all the accommodation costs—this is the multiperiod equivalent of the monetary regime defined in Section 2. In case *2.*, $F$ raises the maximum surplus $\beta(1+\beta)\alpha_F$—the one that makes it indifferent with default—and $M$ picks up the residual accommodation by inflating away the debt $b$ at dates 1 and 2. Finally, there is a threshold for $b$ beyond which the public sector defaults (case *3.*).

There is an asymmetry between $F$ and $M$ because the solvency constraint $b(\beta/p_1+\beta^2/p_2) \leq \sum_{k\geq 0}\beta^k\tau_k$ is linear in surpluses but strictly convex in price levels. Thus $M$ has a unique optimal strategy that consists in setting the

price level at the same level at dates 1 and 2 given that $b_{-1,1} = b_{-1,2} = b$.[26] On the other hand, $F$'s strategy is indeterminate in the sense that all the strategies that deliver the same date-0 present value of surpluses corresponds to an equilibrium. Given its access to perfect bond markets and linear payoff, all that matters is the present value of the future surpluses that $F$ commits to raise.[27]

The following proposition tackles the situation in which neither authority can commit to a policy beyond the current date.

**Proposition 6.** ***($M$ leads, limited commitment)***

1. *If $b \leq \beta^2 \alpha_F p_M$, then, as in the full-commitment case, $M$ can implement the monetary regime $p_t = p_M$ for all $t$ and $F$ accommodates.*

2. *If $\beta^2 \alpha_F p_M < b \leq \alpha_F(p_M + \beta\alpha_M)$, then both $F$ and $M$ partially accommodate. Surpluses at dates 0 and 1 are uniquely determined, and in particular $\tau_1 = 0$ and $\tau_0 > 0$ when $b > \beta^2\alpha_F(p_M + \beta\alpha_M)$. $F$ refinances the date-1 legacy liability $b$ with debt due at date 2, and from date 2 on raises surpluses with date-2 present value $\beta(1+\beta)\alpha_F$. $M$ sets prices $p_1 = p_2 = \min(b/(\beta^2\alpha_F), p_M + \beta\alpha_M)$.*

3. *If $\alpha_F(p_M + \beta\alpha_M) < b \leq \alpha_F(p_M + \alpha_M)$ then in equilibrium $F$ acts as in 2. above, with $\tau_0 = \beta(1+\beta)(1-\beta^2)\alpha_F$. $M$ issues reserves at date 0 in order to purchase date-$t$ outstanding public debt for $t \in \{1; 2\}$ up*

[26] More generally, if $b_{-1,1} \neq b_{-1,2}$, then optimal price-level setting by $M$ implies (ignoring the constraint $p_t \geq p_M$) $p_2/p_1 = \sqrt{(b_{-1,2}/b_{-1,1})}$.

[27] Were its payoff (in the absence of default) strictly concave in surpluses, $F$ would obviously smooth the surpluses by permanently raising its per-period surplus target.

*to a face value equal to* $(b/\alpha_F - \beta\alpha_M)\epsilon/(1+\beta)$ *at each date. M sets* $p_1 = p_2 = b/\alpha_F$ *and* $p_t = p_M$ *for* $t \notin \{1;2\}$.

*4. Otherwise F and M default and play their target at each date.*

**Proof.** Propositions E.3 and E.6 in the online appendix. ■

Limited commitment has two interesting implications. First, $F$ can force $M$ to accommodate at lower debt levels than under full commitment ($\beta^2\alpha_F p_M < \alpha_F p_M$). $F$ does so by kicking the can down the road: It does not raise any surplus at date 1 and rolls over the date-1 liability until date 2. Second, for sufficiently high levels of public debt, $M$ expands its balance sheet. We discuss each implication in turn.

**Strategic fiscal irresponsibility.** By setting the date-1 surplus to 0 and refinancing the date-1 liability with new debt that matures at date 2, $F$ strategically exploits its own time inconsistency in order to force more accommodation by $M$. Since default is a more appealing option when the accommodating effort is more imminent, $F$ makes itself tougher by postponing all consolidation at the latest possible date. As the first instalment moves forward, its current value increases, and fiscal consolidation becomes relatively more costly than default. This forces $M$ to accommodate at date 2 for lower values of $b$ than under full commitment. Anticipating this, $M$ starts accommodating at date 1 so as to optimally smooth price-levels ($p_1 = p_2$) given the strict concavity of the solvency constraint in price levels. As a result, despite being first mover, $M$ starts accommodating before $F$ does.

**Balance-sheet expansion as a commitment to future inflation.** The reason $M$ issues reserves to buy outstanding debt at date 0 when $b > \alpha_F(p_M + \beta\alpha_M)$ is to overcome time inconsistency. At such debt levels, the price levels

$p_1 = p_2$ that are consistent with solvency given $F$'s strategy are ex-post excessive. $M$ would prefer to default at such levels at date 1. To see how balance-sheet expansion serves as a commitment device to ex-post excessive inflation, suppose that $M$ issues (nominal) claims to $r < b$ at dates 1 and 2 to buy back public debt. If $F$ defaults at date 1, $M$ does not default on reserves and sets $p_1 = p_2 = p^d$ such that

$$\frac{(1+\beta)r}{p^d} \leq \epsilon. \tag{16}$$

This means that $M$ is indifferent at date 1 between setting $p_1 = p_2 = p$ and default where

$$(1+\beta)(p - p_M) = \beta(1+\beta)\alpha_M + (1+\beta)(p^d - p_M) \tag{17}$$

This implies that

$$p = \beta\alpha_M + \frac{(1+\beta)r}{\epsilon}, \tag{18}$$

and $M$ can fine tune $r$ so as to reach the desired level $p$.

**A novel rationale for quantitative easing.** Reis (2017) points out that balance-sheet expansions such as the ones that we consider are not immaterial if there is a probability of default on government debt *in equilibrium* whereas remunerated reserves are risk-free. Our strategic approach yields the stronger result that such operations have an impact even when government debt is risk-free in equilibrium. *Swapping two securities that are, along the equilibrium path, perfect substitutes, has an impact on future price lev-*

*els because the out-of-equilibrium possibility of default affects future strategic interactions.*

## 3.3 Fiscal lead

As in the case of monetary lead, we discuss in turn the cases of full and limited commitment.

**Proposition 7.** ***(*$F$ *leads, full commitment)***

1. *If* $b \leq \alpha_F(p_M + \alpha_M)$ *then* $M$ *accommodates as much as possible by setting* $p_1 = p_2 = p_M + \beta\alpha_M$ *and* $p_t = p_M$ *for* $t \notin \{1; 2\}$. $F$ *raises a stream of surpluses with date-0 present value* $(1 + \beta)b/(p_M + \beta\alpha_M)$.

2. *Otherwise* $F$ *and* $M$ *default and play their target at each date.*

**Proof.** Propositions E.2 and E.5 in the online appendix.. ■

The fiscal authority imposes maximum accommodation on the monetary authority and picks up any excess accommodation by raising surpluses (case *1.*), unless $b$ is so large that the public sector defaults (case *2.*). Note that, unlike under monetary lead, there is no pure fiscal regime whereby $F$ sticks to its fiscal target for small levels of $b$. It is an uninteresting consequence of the normalization of this target to 0: $F$ must obviously deviate in order for the public sector to make any nominal repayment at all. With a strictly positive surplus target, the pure fiscal regime would prevail for sufficiently low values of $b$.

The characterization of the equilibrium in the absence of commitment

depends on two endogenous parameters

$$\underline{b} \leq \beta\alpha_F(p_M + \beta\alpha_M) \leq \overline{b} \leq \alpha_F(p_M + \alpha_M)$$

that are defined in the online appendix.

**Proposition 8.** ***(*** $F$ ***leads, limited commitment)***

1. *If* $b \leq \underline{b}$ *then* $M$ *sets* $p_1 = p_M + \beta(1-\beta^2)\alpha_M < p_2 = p_M + \beta(1+\beta)\alpha_M$, *and* $p_t = p_M$ *otherwise, and* $F$ *raises a surplus with sufficient present value and irrelevant timing.*

2. *If* $\underline{b} < b \leq \overline{b}$ *then* $F$ *is inactive at date 0 and subsequently sets surpluses with date-1 present value* $\beta(1+\beta)\alpha_F$. *At date 0,* $M$ *issues reserves to purchase part of the publicly held debt due at date 1. It then sets* $p_1 < p_2 = p_M + \beta(1+\beta)\alpha_M$, *and* $p_t = p_M$ *for* $t \notin \{1;2\}$.

3. *If* $\overline{b} < b \leq \alpha_F(p_M + \alpha_M)$, $F$ *prepays at date 0 a fraction* $1 - \overline{b}/b$ *of dates 1 and 2 liabilities, and then the equilibrium is as in 2. for* $b = \overline{b}$.

4. *Otherwise* $F$ *and* $M$ *default and play their target at each date.*

**Proof.** Propositions E.4 and E.7 in the online appendix. ■

The salient difference between this situation and the three alternative games studied in Propositions 5, 6 and 7 (monetary leadership with and without commitment, and fiscal leadership with commitment) is that $p_1 < p_2$. For $\beta$ close to 1 in particular, $p_1 \simeq p_M$. There is no inflation in the short to medium run (at date 1) and inflation materializes only in the long run (at date 2). This contrasts with equilibrium inflation behavior in the three

alternative games in which it spikes at date 1 and then the price level remains constant. The intuition for this postponement of inflation is as follows. In the absence of commitment, $F$ cannot help imposing maximum monetary accommodation at date 2, forcing a price $p_2 = p_M + \beta(1+\beta)\alpha_M$ such that $M$ is indifferent with default at date 2. This is ex-ante inefficient because it implies that $p_1$ must be set strictly inferior to $p_2$ so that $M$ does not force default at date 1.

The other noteworthy equilibrium features can be found by comparing fiscal and monetary leadership under limited commitment. If $M$ leads, it is only after $F$ has prepaid some liabilities at date 0 and is indifferent with default that $M$ commits to future inflation by expanding its balance sheet (Proposition 6, case *3.*). Conversely, under fiscal leadership, balance-sheet expansion starts at debt levels at which $F$ does not need to enter into fiscal consolidation before the long run (date 2). It is only after $M$ has committed to so much inflation ex-post that it is ex-ante indifferent with default that $F$ starts prepaying debt at date 0 (Proposition 8, case *3.*).

## 3.4 So, who moves first in the US?

A natural empirical counterpart of the exogenous liabilities $b$ that the public sector inherits at date 0 are the public liabilities that resulted from the bailout of many private agents during the 2008 crisis, and from the subsequent attempts at getting the economy out of a liquidity trap. Here we discuss which of the four dynamic models that we considered has the predictions that best fit the evolution of US public finances since 2008. This evolution has three important features:

1. a low realized and expected inflation thus far;

2. a protracted and durable expansion of the Federal Reserve's balance sheet;

3. no discernable attempt at fiscal consolidation.

Regarding fiscal consolidation, to be sure, a standard fiscal-multiplier argument which is absent from our model can rationalize the presence of deficits during the post-2008 recession years—until 2011-12, say. It seems however more challenging to explain persistent realized and projected deficits since 2008 with such a fiscal-multiplier argument.[28] Regarding balance-sheet expansion, the Fed holds both public and private assets, and the latter are absent from our theory. One can argue that the situation would be similar—not politically but in terms of central bank's balance sheet—if the US government had subsidized the owners of these assets and the Fed had acquired the public debt funding such subsidies.

Only the games under limited commitment predict balance-sheet expansion. We argue that the one in which $F$ leads is much better suited to match the three stylized facts above than that of monetary leadership for two reasons. First, if $M$ leads, then it is always the case that inflation realizes in the medium run (date 1). Second, balance-sheet expansion takes place only if the liability shock is so large that date-0 fiscal consolidation is not sufficient to absorb it: No massive asset purchase takes place in the absence of a fiscal consolidation. Conversely, the model with fiscal leadership and limited com-

[28]Comparing data prior to 2000 with post-2008 deficits, Hall (2013) argues that fiscal policy has seriously departed since 2008 from the objective of stabilizing the debt to GDP ratio.

mitment is the only one that allows for the postponement of both inflation and fiscal consolidation to the long run, together with immediate balance-sheet expansion. In other words, our strategic approach offers a narrative of the evolution of US public finances since 2008 as a situation in which the fiscal authority moves first, and forces ex-post excessive accommodation by the central bank, which should ultimately result in inflationary pressure.

## 3.5 Discussion

### 3.5.1 Helicopter money

We focus thus far on open-market operations: The creation of reserves only serves to buy outstanding public debt at the prevailing market price. This is a realistic but arbitrary restriction on $M$'s action set. $M$ could create reserves and directly pay out the proceeds to $F$ at date 0 in order to commit to future inflation—a mechanism deemed "helicopter money". In this situation, reserves are backed only by $M$'s income $\epsilon$ whether $F$ defaults on its obligations or not, and so condition (18) implies that $M$ commits to inflation $p_1 = p_2 = p$ by issuing reserves of size $r$.[29]

The reserves of $M$ are completely unbacked by future taxes along the equilibrium path in the case of helicopter money whereas it is only an out-of-equilibrium threat in the case of open-market operations. Yet the same given quantity $r$ of reserves yields the same commitment to future inflation under open-market-operations as with helicopter money. This implausible result obviously owes to the assumption of purely fixed costs of sovereign default,

[29] Authority $F$ may either rebate the payment from $M$ to the private sector or buy back debt. Only the latter benefits $M$.

which implies in turn that $F$ can credibly threaten with default on 100% of outstanding government debt. Suppose conversely that $F$ cannot credibly threaten $M$ with a default larger than a haircut $h \in (0, 1]$ on government debt. This does not affect the impact of reserves $r$ on inflation in the case of helicopter money since the reserves are fully unbacked anyway. On the other hand, with open-market operations, it amounts to replace $r$ with $rh$ in (16). Thus, the issuance of reserves with face value $r$ yields a date 1, 2 price level

$$p = \beta\alpha_M + \frac{(1+\beta)rh}{\epsilon}, \tag{19}$$

and so for $h$ small balance-sheet expansion only has a limited impact on credible future price levels. In this case, in line with common wisdom, helicopter money is a much more potent inflationary tool than balance-sheet expansion.

### 3.5.2 Strong credibility and politically-insensitive assets

The contrapositive of the result that a highly leveraged central bank can credibly commit to high future inflation is that inflation targeting is more credible ex-ante if the central bank is sufficiently wealthy in all future states of nature, so that it is insensitive to the (out-of-equilibrium) threat of insolvency resulting from an (out-of-equilibrium) fiscal crisis. One way to achieve this is to augment the central bank's mandate with a covenant imposing that it is endowed with an amount of "politically-insensitive" assets that increases in the total liabilities of the public sector—not only the liabilities that are in the bank's balance sheet at a given date. Politically-insensitive assets include gold or foreign assets. In our elementary setup this means committing to an

$\epsilon$ always sufficiently large that debt monetization is never credible.

### 3.5.3 Finite $\epsilon$ and dividend policy

The central bank's dividend policy is immaterial in the limiting case of an arbitrarily small $\epsilon$ studied thus far. The analysis is straightforward when $\epsilon$ is finite. Expression (16) shows that the face value of reserves, $r$, which is necessary to induce a given date-1 price $p_1$ increases with $\epsilon$. Thus $r$ may exceed $b$ for $\epsilon$ sufficiently large in principle, thereby violating our restriction to open-market operations in which public debt and reserves are swapped at fair value. The central bank can easily overcome this by paying out a dividend to the government at date 0, and retaining only a sufficiently small fraction of its income.[30]

### 3.5.4 What if abandoning the currency is also very costly to the fiscal authority?

Finally, we replace Assumption 1 with

**Assumption 2.** *$F$ and $M$ both incur an arbitrarily large disutility from a default on remunerated reserves.*

It seems reasonable to consider the possibility that the government at large incurs large political costs from a currency reform induced by a fiscal crisis. It is easy to see that the analysis is verbatim under fiscal lead. Basically the cost to $F$ from default on reserves are immaterial as $F$, when it leads, never needs to accommodate when reserves are at risk.

[30]This may lead to a negative date-0 book value of the central bank's equity if this future income is not booked at fair value.

Conversely, Assumption 2 gives exorbitant power to $M$ under monetary lead. Suppose that $M$ wants to induce a real surplus $\theta$ from $F$ at date $t$ while staying on target $p_M$. Authority $M$ can simply for example issue unbacked reserves due at date $t$, $r_t$, such that

$$p_M(\epsilon_t + \theta) = r_t, \tag{20}$$

where $\epsilon_t$ is the date-$t$ market value of $M$'s net wealth. This strong ability of the monetary authority to induce fiscal discipline seems highly unrealistic. This casts further doubt on the plausibility of the assumption that $M$ leads.

# 4 Conclusion

This paper applies non-cooperative game theory to the study of the interaction between fiscal and monetary policy. We derive the implications from a wide array of assumptions regarding payoffs, timing, and commitment power. We conclude that the model in which the fiscal authority moves first and the public sector cannot indefinitely commit is the one that delivers the most plausible predictions. We are admittedly only scratching the surface here. There are at least three interesting avenues for future research.

First, the obvious drawback from assuming reduced-form payoffs as we do is that it precludes substantial normative analysis. An interesting research route thus consists in taking a stand on the determinants of the policy targets and on the costs of sovereign default. This would connect equilibrium outcomes to the deep parameters of the economy, thereby paving the road for a mechanism-design approach to a number of important monetary topics.

One can in particular interpret the payoff of the central bank as resulting from a mandate that could be optimally designed. As discussed in Section 3.5, our approach suggests that the actions of the government as shareholder of the central bank have important strategic implications, and so the capitalization of the central bank should be regulated by the mandate ex-ante rather than being mostly left to ex-post negotiations, as is by and large currently the case. Such a micro-founded model could also inform the debate on the opportunity to put the central bank in charge of financial stability and banking supervision, as the bank in this case presumably puts more weight on sovereign default, and this deeply affects its whole equilibrium policies.

Second, we could extend the game to more players along the lines of Aguiar et al. (2015) or Sargent (1986). The "fiscal" authority could explicitly feature executive and legislative branches, and local tax authorities. The lack of cooperation and coordination across these branches could in particular be an endogenous source of commitment power vis-à-vis the monetary authority.

Third, uncertainty is an important feature of the long-term dynamics of public finances that we study, and so departing from perfect foresight is in order. One particularly interesting source of uncertainty regards the "types" of the politicians and central bankers that will be making future decisions—taking into account that politicians are elected and appoint central bankers. The theory of games under incomplete information could in addition be applied to the case in which decision makers privately observe their types and try to signal it to their counterparts and to the private sector.

# References


Aguiar, Mark, Manuel Amador, Emmanuel Farhi, and Gita Gopinath. 2015. "Coordination and Crisis in Monetary Unions." *Quarterly Journal of Economics.* 130 (4): 1727-1779.

Aguiar, Mark, Manuel Amador, Emmanuel Farhi, and Gita Gopinath. 2013. "Crisis and Commitment: Inflation Credibility and the Vulnerability to Sovereign Debt Crises." *Working paper.*

Alesina, Alberto. 1987. "Macroeconomic Policy in a Two-Party System as a Repeated Game." *Quarterly Journal of Economics.* 102: 651-78.

Alesina, Alberto, and Guido Tabellini. 1987. "Rules and Discretion with Non-coordinated Monetary and Fiscal Policy" *Economic Inquiry.* 25: 619-30.

Barthélemy, Jean, Eric Mengus, and Guillaume Plantin. 2018. "Liquidity Demand and Fiscal Arithmetic." *mimeo.*

Bhattarai, Saroj, Gauti Eggertsson, and Bulat Gafarov. 2015. "Time Consistency and the Duration of Government Debt: A Signalling Theory of Quantitative Easing." *NBER Working Paper 21336.*

Cochrane, John. 2005. "Money as Stock." *Journal of Monetary Economics.* 52(3): 501-528.

Dixit, Avinash, and Luisa Lambertini. 2003. "Interactions of Commitment and Discretion in Monetary and Fiscal Policies." *American Economic Review.* 93 (5): 1522-1542.

Hall, Robert. 2013. "Fiscal Stability of High-Debt Nations under Volatile Economic Conditions." *German Economic Review.* 15 (1): 4-22.

Hall, Robert, and Ricardo Reis. 2017. "Achieving Price Stability by Manipulating the Central Bank's Payment on Reserves." *CEPR Discussion Paper 11578.*

Hall, Robert, and Ricardo Reis. 2015. "Maintaining Central-Bank Financial Stability under New-Style Central Banking." *CEPR Discussion Paper 10741.*

Jeanne, Olivier, and Lars Svensson. 2007. "Credible Commitment to Optimal Escape from a Liquidity Trap: The Role of the Balance Sheet of an Independent Central Bank." *American Economic Review.* 97 (1): 474-490.

Kocherlakota, Narayana. 2012. "Central Bank Independence and Sovereign Default." *Banque de France Financial Stability Review.* 16 : 151-154.

Krishnamurthy Arvind, and Annette Vissing-Jorgensen. 2012."The Aggregate Demand for Treasury Debt." *Journal of Political Economy* 120 (2): 233-267

Leeper, Eric. 1991. "Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies." *Journal of Monetary Economics.* 27(1): 129-147.

Reis, Ricardo. 2015. "Different types of central bank insolvency and

the central role of seignorage." *Journal of Monetary Economics*. 73.

Reis, Ricardo. 2017. "QE in the Future: The Central Bank's Balance Sheet in a Fiscal Crisis." *IMF Economic Review*. 65(1): 71-112.

Sargent, Thomas. 1986. "Interpreting the Reagan Deficits." *Federal Reserve Bank of San Francisco, Quarterly Review*.

Sargent, Thomas, and Neil Wallace. 1981. "Some Unpleasant Monetarist Arithmetic." *Quarterly Review, Federal Reserve Bank of Minneapolis.* Issue Fall.

Sims, Christopher. 2013. "Paper Money." *American Economic Review*. 103(2): 563-84.

Svensson, Lars. 2017. "The Relation between Monetary Policy and Financial-Stability Policy," paper presented at the XXI Annual Conference of the Central Bank of Chile, "Monetary Policy and Financial Stability: Transmission Mechanisms and Policy Implications," Santiago, Chile, November 16-17, 2017.

Tabellini, Guido. 1986. "Money Debt and Deficits in a Dynamic Game." *Journal of Economic Dynamics and Control*. 10. 427-442.

# Appendix

## Proof of Proposition 1

**Step 1: Pure-strategy equilibria are either all default-free or all feature default.** If a strategy profile $(\tau, p) \neq (\tau_F, p_M)$ constitutes an equilibrium then it must be that $b \geq \tau p$ otherwise at least one player could strictly benefit from getting closer to target. If $b = \tau p$ then it must be that

$$g'_F(\tau - \tau_F) \leq f'_F(0), \tag{21}$$

$$g'_M(p - p_M) \leq \frac{b}{p^2} f'_M(0). \tag{22}$$

If either inequality is not satisfied, the associated player could benefit from getting closer to target. Conversely if $b > \tau p$ then none of these inequalities holds, otherwise the associated player would benefit from accommodating more.

Suppose $b \leq \tau_M p_F$ and consider an equilibrium $(\tau, p)$. If $\tau > \tau_M$ then $p \leq p_F$ and $b/p^2 \geq \tau/p > \tau_M/p$, and so (21) is not satisfied whereas (22) is, a contradiction. Thus $\tau \leq \tau_M$, (22) is satisfied and hence any equilibrium must be such that $b = \tau p$. If $b > \tau_M p_F$, then an equilibrium cannot be such that $b = \tau p$ otherwise (21) and (22) would imply $b \leq \tau_M p_F$.

**Step 2: Default-free equilibria.** Suppose $b \leq \tau_M p_F$. Define

$$\tau(b) = \inf \left\{ \tau \in [\tau_F, \tau_M] \mid \frac{b}{\tau} g'_M \left( \frac{b}{\tau} - p_M \right) \leq \tau f'_M(0) \right\}, \tag{23}$$

$$b^1 = \sup\{b \geq \tau_F p_M \mid \tau(b) = \tau_F\}, \tag{24}$$

$$b^2 = \tau_M p_M, \tag{25}$$

$$b_- = \inf\{b^1; b^2\}, \tag{26}$$

$$b_+ = \sup\{b^1; b^2\}. \tag{27}$$

A profile $(\tau, p)$ is an equilibrium if and only if $\tau \geq \tau_F$, $p \geq p_M$, $\tau p = b$, and $(\tau, p)$ satisfies (21) and (22). The equilibria are therefore exactly $(\tau, b/\tau)$ for $\tau \in [\tau(b), \inf\{b/p_M; \tau_M\}]$, and so they can be described as in the proposition.

**Step 3: Equilibrium with default.** Suppose $b > \tau_M p_F$. For a level of default $d > 0$, the first-order conditions

$$g'_F(\tau - \tau_F) = f'_F(d), \tag{28}$$

$$g'_M(p - p_M) = \frac{b}{p^2} f'_M(d) \tag{29}$$

implicitly define $\tau(d)$ and $p(d, b)$ as strictly increasing functions. Furthermore, (29) implies that $b/p(d, b)$ is strictly increasing in b. Since $d(\tau, p, b) = b/p - \tau$ is strictly decreasing in $p$ and $\tau$,

$$d = \frac{b}{p(d, b)} - \tau(d) \tag{30}$$

admits a unique solution $d$ for $b$ fixed. This solution characterizes the unique equilibrium. Finally, the comparative statics with respect to $b$ are a straight-

forward consequence from the fact $b/p(d,b)$ is strictly increasing in b holding $d$ fixed.

## Proof of Proposition 2

**Step 1: Equilibrium existence and uniqueness.** The payoffs $E[U_X(\tau,p)]$ for $X \in \{F;M\}$ are strictly concave, bounded above, and differentiable and so $(\tau,p)$ is a Nash equilibrium if and only if

$$\frac{\partial E[U_F(\tau,p)]}{\partial \tau} = \frac{\partial E[U_M(\tau,p)]}{\partial p} = 0. \tag{31}$$

A similar reasoning to that in Step 3 in the proof of Proposition 1 shows that for every $\sigma > 0$, this system has a unique solution $(\tau(\sigma), p(\sigma))$.

**Step 2: A simple characterization of convergence.** Suppose that if any two sequences of strictly positive real numbers $(\sigma_n)_{n\geq 0}$ and $(\sigma'_n)_{n\geq 0}$ are such that $(\sigma_n, \tau(\sigma_n), p(\sigma_n)) \to (0,\tau,p)$ and $(\sigma'_n, \tau(\sigma'_n), p(\sigma'_n)) \to (0,\tau',p')$, then it must be that $(\tau,p) = (\tau',p')$. This implies that $\lim_{\sigma\downarrow 0}(\tau(\sigma), p(\sigma))$ exists and is equal to $(\tau,p)$.

**Proof.** Suppose otherwise. Then there exists a sequence $(\sigma_n)_{n\geq 0} \downarrow 0$ such that $(\tau(\sigma_n), p(\sigma_n))$ is bounded away from $(\tau,p)$. This latter sequence is bounded, however, and so it admits a convergent subsequence which by assumption must converge to $(\tau,p)$, a contradiction.

**Proof of the proposition.** Consider then a sequence $(\sigma_n)_{n\geq 0} > 0$ such that $(\sigma_n, \tau(\sigma_n), p(\sigma_n)) \to (0,\tau,p)$. It must be that $\tau p \leq b$, and our regularity

assumptions imply that

$$\Omega = \lim_{n\to+\infty} P\left(\epsilon_F + \epsilon_M < \frac{\ln \frac{b}{\tau p}}{\sigma_n}\right) \tag{32}$$

exists and satisfies

$$0 = \lim_{n\to+\infty} \frac{\partial E[U_F(\tau(\sigma_n), p(\sigma_n))]}{\partial \tau} = -g'_F(\tau - \tau_F) + \Omega f'_F\left(\frac{b}{p} - \tau\right), \tag{33}$$

$$0 = \lim_{n\to+\infty} \frac{\partial E[U_M(\tau(\sigma_n), p(\sigma_n))]}{\partial p} = -g'_M(p - p_M) + \Omega \frac{b}{p^2} f'_M\left(\frac{b}{p} - \tau\right). \tag{34}$$

If $b > \tau p$, then $\Omega = 1$ and $\{33; 34\}$ imply that $(\tau, p)$ is the same as in the unique equilibrium with default under certainty. It must therefore be that $b > \tau_M p_F$. Otherwise, $b = \tau p$ and hence $\{33; 34\}$ implies that $b \leq \tau_M p_F$ by definition of $(\tau_M, p_F)$.

The limit $(\tau, p)$ is therefore unique for all such converging sequences, and defined as in the proposition. Step 2 thus implies that $(\tau(\sigma), p(\sigma))$ converges as stated in the proposition.

## Proof of Proposition 3

Given the fixed default costs, a strategy profile $(\tau, p)$ is an equilibrium if and only if either $\tau p = b$ or $(\tau, p) = (\tau_F, p_M)$. Default-free equilibria thus correspond to the portion of the hyperbolae $\tau p = b$ in the plane $(\tau, p)$ that is within the rectangle $[\tau_F, \tau_F + \alpha_F] \times [p_M, p_M + \alpha_M]$. Default is an equilibrium outcome if and only if $b \geq \tau_F(p_M + \alpha_M)$ and $b \geq (\tau_F + \alpha_F)p_M$, otherwise one authority always finds accommodation preferable to default.

# Fiscal and Monetary Regimes: A Strategic Approach

by Jean Barthélemy and Guillaume Plantin

## Not for publication - Online appendix

Section 3 in the paper studies four games resulting from the combination of two possible types of strategic interactions between $M$ and $F$—fiscal *versus* monetary lead on one hand, and full *versus* limited commitment on the other hand. This online appendix adds a third variation to the game—unanticipated versus anticipated fiscal shock. We first study a version of the game in which the exogenous liabilities $b$ due at dates 1 and 2 are *unanticipated*: $F$ and $M$ discover them at date 1. We then study the situation in the paper in which these liabilities are *anticipated*: As in the paper, $F$ and $M$ discover them at the outset of date 0. The unanticipated version is a useful stepping stone to solve for the anticipated one that we consider in the paper. We present the results under the assumption that $\beta$ is sufficiently close to 1, in a sense made precise in Section E.3.

These three different variations imply that we consider $2^3 = 8$ possible games. The following Section E.1 first studies the situations of unanticipated fiscal shocks. Section E.2 then tackles that in which the shocks are anticipated at date 0. We postpone all the proofs to Section E.3.

# E.1 Unanticipated fiscal shocks

In this Section E.1, the public sector discovers the outstanding liabilities $b_{-1,1} = b_{-1,2} = b$ at the outset of date 1. This implies that they played their respective policy targets and did not issue claims at date 0. We first characterize the equilibrium when parties can commit to any future action plan, and then we do so when they cannot.

## E.1.1 Full commitment

We characterize in turn equilibria under monetary and fiscal lead.

**Proposition E.1.** ***(Unanticipated shocks, full commitment,*** $M$ ***leads)***

1. *If* $b \leq \beta\alpha_F p_M$ *then* $M$ *announces* $p_t = p_M$ *for all* $t$*, and the fiscal authority raises a stream of surpluses with date-1 present value* $(1+\beta)b/p_M$*.*

2. *If* $\beta\alpha_F p_M < b \leq \beta\alpha_F(p_M+\beta\alpha_M)$*, then* $M$ *announces* $p_1 = p_2 = b/\beta\alpha_F$*,* $p_t = p_M$ *for* $t > 2$*, and* $F$ *raises surpluses with date-1 present value* $\beta(1+\beta)\alpha_F$*.*

3. *Otherwise* $F$ *and* $M$ *default and play their target at each date.*

**Proof.** See Section E.3. ■

**Proposition E.2.** ***(Unanticipated shocks, full commitment,*** $F$ ***leads)***

1. *If* $b \leq \beta\alpha_F(p_M + \beta\alpha_M)$ *then* $F$ *raises a stream of surpluses with date-1 present value* $(1+\beta)b/(p_M + \beta\alpha_M)$*, and* $M$ *sets* $p_1 = p_2 = p_M + \beta\alpha_M$ *and* $p_t = p_M$ *for* $t \notin \{1; 2\}$*.*

2. *Otherwise* $F$ *and* $M$ *default and play their target at each date.*

**Proof.** See Section E.3. ■

### E.1.2 Limited commitment

Again, we consider first monetary then fiscal lead. For conciseness, Proposition E.3 below describes the equilibrium provided $\beta$ is sufficiently large holding all parameters other than $b$ fixed.[1]

**Proposition E.3.** ***(Unanticipated shocks, limited commitment,*** $M$ ***leads)***

1. *If* $b \leq \beta^2\alpha_F p_M$*, then, as in the full-commitment case,* $M$ *can implement the monetary regime* $p_t = p_M$ *for all* $t$*.*

2. *If* $\beta^2\alpha_F p_M < b \leq \beta^2\alpha_F(p_M + \beta\alpha_M)$*, then at date 1* $F$ *sets* $\tau_1 = 0$ *and refinances the date-1 liability with debt due at date 2. Afterwards, it raises surpluses with date-2 present value* $\beta(1+\beta)\alpha_F$*, and so is indifferent between defaulting or not at date 2. Authority* $M$ *sets* $p_1 = p_2 = b/(\beta^2\alpha_F) > p_M$ *and* $p_t = p_M$ *otherwise.*

[1] The exact condition on $\beta$ is stated in the proof of Proposition E.3, which also explains how the results are modified for lower values of $\beta$. All insights carry over.

*3. Otherwise the public sector defaults.*

**Proof.** See Section E.3. ■

**Proposition E.4.** ***(Unanticipated shocks, limited commitment,*** *$F$* ***leads)***

1. *If $b \leq \underline{b} < \beta\alpha_F(p_M + \beta\alpha_M)$, then $M$ sets $p_1 = p_M + \beta(1-\beta^2)\alpha_M < p_2 = p_M + \beta(1+\beta)\alpha_M$, and $p_t = p_M$ for $t > 2$. $F$ raises a surplus with date-1 present value $b(1/p_1 + \beta/p_2)$.*

2. *Otherwise the public sector defaults.*

**Proof.** See Section E.3. ■

**Remark on the value of commitment.** It is easy to see that the equilibria under full commitment described in Propositions E.1 and E.2 still obtain under the weaker assumption that only the leader ($M$ in Proposition E.1 and $F$ in Proposition E.2) can commit. This would however not be true if the duration of the inherited liabilities, measured for example by $b_{-1,2}/b_{-1,1}$, was sufficiently large. In this case, the follower could find a given date-2 accommodating effort preferable to default at date 1, but no longer so at date 2 once this effort is no longer discounted by $\beta$. The simplifying assumption that $b_{-1,1} = b_{-1,2} = b$ enabled us to abstract from this time-inconsistency problem in this case of unanticipated shocks. The case of anticipated shocks studied in the following section is precisely meant to study this time inconsistency between dates 0 and 1.

## E.2 Anticipated fiscal shocks

Consider now the situation studied in the paper in which the public sector learns at the outset of date 0 that it inherits $b_{-1,1} = b_{-1,2} = b$. The case in which parties can fully commit is a straightforward adaptation of that of full commitment and unanticipated shocks:

**Proposition E.5.** ***(Anticipated shocks, full commitment)*** *Under full commitment, the respective equilibria under monetary and fiscal lead are respectively described by Proposition E.1 and E.2 up to the substitution of $\{\alpha_M; \alpha_F\}$ with $\{\alpha_M/\beta; \alpha_F/\beta\}$.*

**Proof.** See Section E.3. ■

Intuitively, under full commitment, date-0 actions are immaterial. The schedule under which $F$ collects surpluses is irrelevant, only their present value matters, and so assuming $\tau_0 = 0$ is without loss of generality. (This is only true under full commitment as we shall see below). Regarding $M$, there are no liabilities at date 0 and thus no gains from setting $p_0 > p_M$. The only difference with the unanticipated case stems from the fact that at date 0, $F$ and $M$ discount with $\beta$ the date-1 present value of their respective costs of accommodations, whereas the cost of default along the equilibrium path is unchanged for them. Default is therefore relatively more costly than accommodation to the public sector at date 0 when the fiscal shock is more remote than at date 1. This induces time-inconsistency: $F$ and $M$ commit to accommodating higher levels of debt at date 0 than at date 1.

We now study in turn monetary and fiscal leads under limited commitment in this context of anticipated shocks.

**Proposition E.6.** ***(Anticipated shocks, limited commitment,*** $M$ ***leads)***

1. *If* $b \leq \beta^2\alpha_F(p_M + \beta\alpha_M)$ *then the equilibrium is as in the case of an unanticipated shock described in Proposition E.3.*

2. *If* $\beta^2\alpha_F(p_M + \beta\alpha_M) < b \leq \alpha_F(p_M + \beta\alpha_M)$ *then* $F$ *prepays at date 0 shares of* $b_{-1,1}$ *and* $b_{-1,2}$*, each with face value* $b - \beta^2\alpha_F(p_M + \beta\alpha_M)$*.* $M$ *is inactive at date 0. The equilibrium at subsequent dates is described in Proposition E.3 in the case* $b = \beta^2\alpha_F(p_M + \beta\alpha_M)$*.*

3. *If* $\alpha_F(p_M + \beta\alpha_M) < b \leq \alpha_F(p_M + \alpha_M)$ *then* $M$ *issues reserves at date 0 in order to purchase date-t outstanding public debt for* $t \in \{1; 2\}$ *up to a face value equal to* $(b/\alpha_F - \beta\alpha_M)\epsilon/(1 + \beta)$ *at each date.* $M$ *sets* $p_1 = p_2 = b/\alpha_F$ *and* $p_t = p_M$ *for* $t \notin \{1; 2\}$*.* $F$ *raises surpluses*

$$\begin{cases} \tau_0 = \beta(1 + \beta)(1 - \beta^2)\alpha_F, \\ \tau_2 = \beta(1 + \beta)\alpha_F, \\ \tau_t = 0 \text{ for } t \notin \{0; 2\}. \end{cases} \tag{E.1}$$

4. *Otherwise* $F$ *and* $M$ *default and play their target at each date.*

**Proof.** See Section E.3. ■

These results come from the time-inconsistency problem induced by the fact that default along the equilibrium path comes at the same current cost no matter how remote it is. This implies that the only difference between the unanticipated and anticipated cases is that the public sector is more willing to avert default at date 0 than at date 1. If $b$ is sufficiently small that the public sector does not default in the unanticipated case (case *1.*), then the date-0 anticipation of the shock is immaterial. Beyond this threshold, $F$ and $M$ can use date-0 actions to overcome their time-inconsistency problem. $F$ buys back some of the outstanding debt at date 0. $M$ can commit to more inflation than it would find ex-post optimal with the creation of reserves backed by public debt which would threaten its solvency in case of sovereign default.

As in the static case (or in the case of full commitment), the leader $M$ accommodates at date 0—that is, issues reserves to commit to future inflation—only if $b$ is such that $F$ has accommodated as much as possible and is indifferent with default (case *3.*). Before this, $F$ is the only date-0 mover and bears the entire burden of accommodation by buying back debt.

Finally, Proposition E.7 studies the case of fiscal lead. For brevity we state it assuming $\beta(1+\beta) \geq 1$—arguably the most plausible case, and discuss in the proof of the proposition how $\beta(1+\beta) < 1$ (slightly) affects the results.

**Proposition E.7.** ***(Anticipated shocks, limited commitment, $F$ leads)*** *There exists $\underline{b}$, $\overline{b}$ such that:*

1. *If $b \leq \underline{b}$ then the equilibrium is as in the case of an unanticipated shock described in Proposition E.4.*

2. *If $\underline{b} < b \leq \overline{b}$ then $F$ is inactive at date 0 and subsequently sets surpluses with date-1 present value $\beta(1+\beta)\alpha_F$. At date 0, $M$ issues reserves to purchase part of the publicly held debt due at date 1. It then sets $p_1 \in (p_M + \beta(1-\beta^2)\alpha_M, p_M + \beta(1+\beta)\alpha_M)$, $p_2 = p_M + \beta(1+\beta)\alpha_M$, and $p_t = p_M$ for $t \notin \{1;2\}$.*

3. *If $\overline{b} < b \leq \alpha_F(p_M + \alpha_M)$, $F$ prepays at date 0 a fraction $1 - \overline{b}/b$ of dates 1 and 2 liabilities, and then the equilibrium is as in 2. for $b = \overline{b}$.*

4. *Otherwise $F$ and $M$ default and play their target at each date.*

**Proof.** See Section E.3. ■

Again, if $b$ is sufficiently small that the public sector would not default in the unanticipated case, then anticipation plays no role (case *1.*). Otherwise, the follower $M$ is the only accommodating party at date 0 until its commitment to future inflation makes it indifferent with default. $F$ then picks up the residual accommodation need up to the point at which the public sector finds default ex-ante optimal.

In sum, Propositions E.6 and E.7 show that debt prepayment and remunerated reserves are commitment devices that enable the public sector to accommodate a fiscal shock as long as it finds it optimal to do so under full commitment ($b \leq \alpha_F(p_M + \alpha_M)$ from Proposition E.5) provided the shock is anticipated. Whether $F$ and $M$ can commit or not affects how they share the accommodation effort, however.

## E.3 Proofs

### Proof of Proposition E.1

Suppose $M$ plays $p_t = p_M$ for all $t \geq 1$. Default grants $F$ a payoff $-\beta(1+\beta)\alpha_F$. $F$ may instead accommodate, which it can do in infinitely many ways as long as the stream of surpluses that it raises has date-1 present value $(1+\beta)b/p_M$. Therefore, as long as $b \leq \beta\alpha_F p_M$, $M$ optimally commits to the profile $p_t = p_M$ for all $t \geq 1$ and $F$ accommodates.

If $b > \beta\alpha_F p_M$, then $M$ can avoid default with any strategy such that $p_t = p_M$ for $t \geq 3$ and $b/p_1 + \beta b/p_2 \leq \beta(1+\beta)\alpha_F$. Setting $p_1 = p_2 = b/(\beta\alpha_F)$ maximizes its payoff over these strategies, and so $M$ plays this as long as $V_1^M = -(1+\beta)(b/(\beta\alpha_F) - p_M) \geq -\beta(1+\beta)\alpha_M$, or $b \leq \beta\alpha_F(p_M + \beta\alpha_M)$.

The public sector defaults otherwise.

### Proof of Proposition E.2

Suppose first that $F$ not only raises surpluses and issues debt, but also sets $p_1$ and $p_2$ subject to $p_1 + \beta p_2 \leq (1+\beta)(p_M + \beta\alpha_M)$. In this case, the best strategy that does not involve default consists in minimizing the real burden of debt $b/p_1 + \beta b/p_2$ by setting $p_1 = p_2 = p_M + \beta\alpha_M$, and in raising matching surpluses with date-1 present value $(1+\beta)b/(p_M + \beta\alpha_M)$. This is preferable to default if and only if $b \leq \beta\alpha_F(p_M + \beta\alpha_M)$.

For such values of $b$, $F$ can indeed induce $M$ to set $p_1 = p_2 = p_M + \beta\alpha_M$ by announcing any strategy such that the public sector is exactly solvent with such prices $p_1$ and $p_2$ (and $p_t = p_M$ for $t \geq 3$). Authority $F$ can for example set $\tau_1 = \tau_2 = b/(p_M + \beta\alpha_M)$, $b_{t,t'} = 0$ for all $t' > t \geq 1$. The monetary authority then finds it optimal to accommodate with $p_1 = p_2 = p_M + \beta\alpha_M$.

## Proof of Proposition E.3

We derive the equilibrium by backward induction.

Suppose that the public sector has to repay $b'$ at date 2. From Proposition 4, $M$ chooses $p_2 = p_M$ as long as $F$ can bear the accommodation cost ($b' \leq \beta(1+\beta)\alpha_F p_M$). For $b' \in [\beta(1+\beta)\alpha_F p_M, \beta(1+\beta)\alpha_F(p_M + \beta(1+\beta)\alpha_M)]$, $M$ forces $F$ to accommodate as much as possible ($\tau_2 = \beta(1+\beta)\alpha_F$) and $M$ deviates from target ($p_2 = b'/(\beta(1+\beta)\alpha_F)$). For larger values of $b'$, the public sector defaults.

We now consider the problem from the date-1 perspective. Suppose $M$ has played and $p_1$ is fixed. Authority $F$ optimally seeks to maximize $p_2$ in order to minimize the real debt burden. From the date-2 analysis, $p_2$ is weakly increasing in $b'$, and $b' = b + p_2(b/p_1 - \tau_1)/\beta$ is in turn increasing in $p_2$ and decreasing in $\tau_1$. We now prove a series of auxiliary results.

*Result 1. Authority $M$ can set $p_1 = p_2 = p_M$ if and only if $b \leq \beta^2\alpha_F p_M$.*
**Proof.** If $b \leq \beta^2\alpha_F p_M$, then $b'/p_2 \leq b/p_M + b/(\beta p_M) \leq \beta(1+\beta)\alpha_F$, and so from the date-2 analysis $M$ can set $p_1 = p_2 = p_M$. Conversely if $b > \beta^2\alpha_F p_M$ then if $M$ sets $p_1 = p_M$, $F$ sets $\tau_1 = 0$ and rolls over date-1 debt so that $b' = b + p_2 b/(\beta p_M) > \beta(1+\beta)\alpha_F$, implying that $M$ must accommodate at date 2. ■

*Result 2. If $b > \beta^2\alpha_F p_M$, then $\tau_2 = \beta(1+\beta)\alpha_F$ in equilibrium.*
**Proof.** If $\tau_2 < \beta(1+\beta)\alpha_F$, then it must be that $p_2 = p_M$ otherwise $M$ would induce a higher $\tau_2$ with a lower price level $p_2$. From Result 1, it must then be that $p_1 > p_M$. This also implies that $\tau_1 > 0$, otherwise $M$ could reduce $p_1$ and induce more accommodation at date 1 or/and 2. Furthermore, it must be that this strategy dominates the strategy $(\tau_1 = 0, \tau_2 = \beta(1+\beta)\alpha_F)$, i.e.,

$$b\left(\frac{\beta}{p_M} + \frac{1}{p_1}\right) \leq \beta^2(1+\beta)\alpha_F. \tag{E.2}$$

But inequality (E.2) implies that $M$ can reduce $p_1$ (possibly simultane-

ously increasing $p_2$ if (E.2) binds) and induce more accommodation by $F$ this way.

*Result 3. If $b > \beta^2\alpha_F p_M$, then if an equilibrium is such that $\tau_1 > 0$ it is also such that $p_2 = p_M + \beta(1+\beta)\alpha_M$.*

**Proof.** From Result 2, $\tau_2 = \beta(1+\beta)\alpha_F$. So if $p_2 < p_M + \beta(1+\beta)\alpha_M$, $F$ can reduce $\tau_1$ to increase $b'$ thereby forcing $M$ to raise $p_2$ unless $\tau_1 = 0$. ■

Suppose $b > \beta^2\alpha_F p_M$. These results imply that if $M$ seeks to implement an equilibrium such that $p_1 = p_2$ ("smoothing strategy") then it must be such that $\tau_1 = 0$, $\tau_2 = \beta(1+\beta)\alpha_F$, and so $p_1 = p_2 = b/(\beta^2\alpha_F)$. Such a smoothing strategy grants $M$ a payoff (viewed from date 1)

$$(1+\beta)\left(\frac{b}{\beta^2\alpha_F} - p_M\right), \tag{E.3}$$

and is thus feasible as long as $b \leq \beta^2\alpha_F(p_M + \beta\alpha_M)$.

Alternatively, if $M$ prefers to induce $\tau_1 > 0$ ("extracting strategy"), then it must sacrifice inflation smoothing by setting $p_1 < p_2 = p_M + \beta(1+\beta)\alpha_M$. For a given $b$ the date-1 price level $p_1(b)$ solves in this case

$$\frac{b}{p_1(b)} + \frac{\beta b}{p_M + \beta(1+\beta)\alpha_M} = \beta(1+\beta)\alpha_F, \tag{E.4}$$

or $p_1(b) = p_M$ if (E.4) admits no solution larger than $p_M$. This extracting strategy can be sustained up to the point at which $p_1(b) = p_M + \beta(1-\beta^2)\alpha_M$ at which $M$ is indifferent with default. This defines an upper bound $\underline{b}$ for the values of $b$ such that this extracting strategy is admissible:

$$\underline{b} = \frac{\beta\alpha_F[p_M + \beta(1+\beta)\alpha_M][p_M + \beta(1-\beta^2)\alpha_M]}{p_M + \beta[1+\beta(1-\beta)]\alpha_M}. \tag{E.5}$$

The smoothing strategy is clearly dominant for $b$ in a right neighborhood of $\beta^2\alpha_F p_M$ as it converges to the commitment solution whereas the extracting one does not. Furthermore, straightforward computations show that $\underline{b} < \beta^2\alpha_F(p_M + \beta\alpha_M)$ for $\beta$ sufficiently close to 1, and so smoothing also dominates in the left-neighborhood of $\beta^2\alpha_F(p_M + \beta\alpha_M)$ for such values of $\beta$. Routine computations show that the slope of the payoff of the smoothing strategy as a function of $b$ is smaller than that of the extracting strategy over $[\beta^2\alpha_F p_M, \underline{b}]$ for $\beta$ sufficiently large. This implies that the smoothing strategy always

dominates the extracting one in this case, and establishes the proposition.

For lower values of $\beta$, $\underline{b} > \beta^2\alpha_F(p_M + \beta\alpha_M)$. This implies that the equilibrium features an additional region that stands between the smoothing-strategy region and the default region in which $M$ implements the extracting strategy.

## Proof of Proposition E.4

Authority $F$ ex-post optimally imposes full accommodation by $M$ at date 2 unless the public sector defaults, and so $p_2 = p_M + \beta(1+\beta)\alpha_M$ in the absence of default. This implies that at date 1, $F$ forces $M$ to the maximum feasible accommodation given date-2 actions. The constraint $(p_1 - p_M) + \beta(p_2 - p_M) \leq \beta(1+\beta)\alpha_M$ implies $p_1 = p_M + \beta(1-\beta^2)\alpha_M$ in the absence of default. Authority $F$ then raises a stream of surplus with date-1 present value $b(1/p_1 + \beta/p_2)$ as long as this accommodation cost is lower than $\beta(1+\beta)\alpha_F$, and otherwise the public sector defaults. The default cutoff is $\underline{b}$ defined in equation (E.5).

## Proof of Proposition E.5

Under commitment, there is nothing relevant that either authority can do at date 0 that it cannot do later on. Setting $p_0 > p_M$ is useless because there is no outstanding public debt at date 0, and the timing of surpluses is immaterial. Furthermore, the scaling of defaults costs by $1/\beta$ stems from the assumption that $F$ and $M$ incur these costs immediately given a non-sustainable path for public finances, as formalized in (13) and (14).

## Proof of Proposition E.6

Suppose first that $b \leq \beta\alpha_F^2(p_M + \beta\alpha_M)$. For such values of $b$, Proposition E.3 shows that there is no default and characterizes the equilibrium in the case of an unanticipated fiscal shock and limited commitment. There is nothing that either party can do at date 0 that would improve its situation over this unanticipated case. First, as is shown below, issuing reserves can only lead $M$ to set higher future price levels than in the absence of reserves. Second, pre-paying debt by raising $\tau_0 > 0$ would also be costly to $F$ as it would enable $M$ to reduce $p_1$ and $p_2$. The equilibrium is therefore as described in Proposition E.3.

Consider then the case $b \in (\beta^2\alpha_F(p_M + \beta\alpha_M), \alpha_F(p_M + \beta\alpha_M)]$. For such values of $b$, Proposition E.3 shows that there is default in the absence of commitment if the shocks are unanticipated, whereas Proposition E.5 shows that the equilibrium under commitment does not feature default when shocks are anticipated as of date 0. In this case, $M$ can let $F$ bear the brunt of ex-ante accommodation by being inactive at date 0. The fiscal authority then optimally prepays the minimum amount of future debt at date 0 so as to elicit maximum accommodation by $M$ at dates 1 and 2 ($p_1 = p_2 = p_M + \beta\alpha_M$). This implies reducing future outstanding liabilities at dates 1 and 2 from $b$ to $\beta^2\alpha_F(p_M + \beta\alpha_M)$. This requires raising $\tau_0 = \beta(1+\beta)[b/(p_M + \beta\alpha_M) - \beta^2\alpha_F]$. Given that $F$ also raises $\tau_2 = \beta(1 + \beta)\alpha_F$, it is willing to bear all the brunt of ex-ante accommodation this way as long as

$$\tau_0 + \beta^2\tau_2 \leq \beta(1 + \beta)\alpha_F, \tag{E.6}$$

or

$$b \leq \alpha_F(p_M + \beta\alpha_M). \tag{E.7}$$

If $b \in (\alpha_F(p_M+\beta\alpha_M), \alpha_F(p_M+\alpha_M)]$, then again the public sector defaults in the absence of commitment if the fiscal shock is unanticipated (Proposition E.3) whereas it prefers to make good under commitment if it anticipates the shock at date 0 (Proposition E.5). The ex-ante accommodation effort of $F$ described in the previous case is no longer sufficient, however, and so $M$ must also contribute by committing to price levels $p_1$ and $p_2$ such that $\beta(1 + \beta)\alpha_F \geq \beta(b/p_1 + \beta b/p_2)$. It does so at the lowest cost by committing to $p_1 = p_2 = b/\alpha_F$.

We now show that $M$ can commit to such future price levels by issuing remunerated reserves. Suppose M issues reserves such that $r_{0,1} = r_{0,2} = r$. In case of sovereign default at date 1, $M$ must set $p_1^d$ at date 1 and $p_2^d$ at date 2 such that

$$\frac{r}{p_1^d} + \frac{\beta r}{p_2^d} \leq \epsilon, \tag{E.8}$$

where $\epsilon$ is the date-1 present value of its (real) income. Authority $M$ maximizes its payoff by doing so with $p_1^d = p_2^d = (1 + \beta)r/\epsilon$. This implies that $M$

is at date 1 indifferent between any prices $p_1$ and $p_2$ that avert default and defaulting if and only if

$$p_1 + \beta p_2 = \beta(1+\beta)\alpha_M + p_1^d + \beta p_2^d, \tag{E.9}$$

and so $p_1 = p_2 = b/\alpha_F$ is feasible for $r = \epsilon(b/\alpha_F - \beta\alpha_M)/(1+\beta)$.

Finally, if $b > \alpha_F(p_M + \alpha_M)$, then the public sector defaults as it does under full commitment.

## Proof of Proposition E.7

The proof mirrors that of Proposition E.6 and we only sketch it.

First, if $b \leq \underline{b}$, where $\underline{b}$ is defined in (E.5), then $F$ and $M$ do not default when the fiscal shock is unanticipated. For the same reasons as when $M$ leads, they both are inactive at date 0 and then play the equilibrium described in Proposition E.4.

Second, there exists $\overline{b} > \underline{b}$ defined below such that for $b \in (\underline{b}, \overline{b}]$, $F$ remains inactive at date 0 but induces $M$ to commit to higher price levels than it finds ex-post optimal so that the public sector does not default. The difference with the situation in which $M$ leads is that $p_1 < p_2$ ex-post, and so $M$ only raises $p_1$ to a level $p_1(b)$ such that

$$\frac{b}{p_1(b)} + \frac{\beta b}{p_M + \beta(1+\beta)\alpha_M} = \beta(1+\beta)\alpha_F. \tag{E.10}$$

The threshold $\overline{b}$ is implicitly defined as

$$p_1(\overline{b}) - p_M + \beta^2(1+\beta)\alpha_M = (1+\beta)\alpha_M. \tag{E.11}$$

Note that $\beta(1+\beta) \geq 1$ ensures that $p_1(\overline{b}) < p_2 = p_M + \beta(1+\beta)\alpha_M$ and so $M$ never fully smoothes inflation. In order to commit to the price level $p_1(b)$, $M$ issues a date-1 reserve $r = \epsilon[p_1(b) - \beta(1-\beta^2)\alpha_M]$. This way it commits to a date-1 price level in case of default $p_1^d$ such that $p_1^d\epsilon = r$. This means that along the equilibrium path in which public debt is fully honored, $F$ can extract a maximum price level $p_1$ such that

$$p_1 + \beta p_2 = \beta(1+\beta)\alpha_M + p_1^d + \beta p_M, \tag{E.12}$$

or

$$p_1 = \beta(1-\beta^2)\alpha_M + \frac{r}{\epsilon} = p_1(b). \tag{E.13}$$

If $\beta(1+\beta) < 1$, then the value of $p_1$ that $M$ commits to reaches $p_2$ before $M$'s indifference with default as $b$ increases. Thus, an additional region of $b$ arises between cases *2.* and *3.*, whereby $M$ uses reserves to commit to higher values of both $p_1$ and $p_2$, with $p_1 = p_2$.

For $b \in (\overline{b}, \alpha_F(p_M + \alpha_M)]$, $F$ must also accommodate at date 0. It does so by prepaying a (nominal) amount $b - \overline{b}$ of public debt at each date 1,2 and then lets $M$ accommodate as above for $b = \overline{b}$.

Otherwise the public sector defaults.